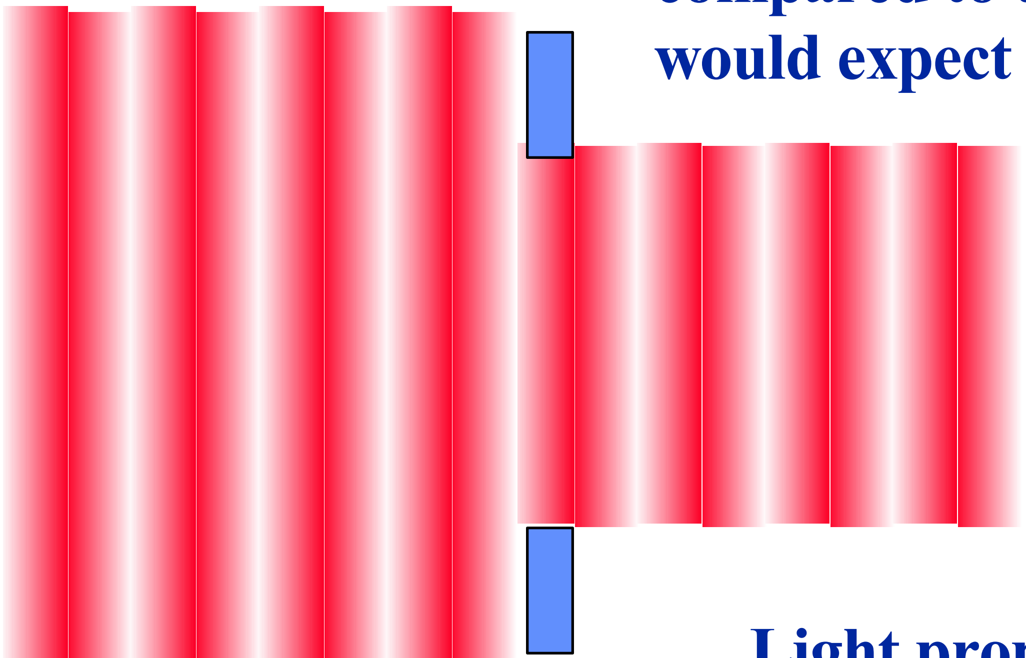
Interference and Diffraction

Chapter 35-36

Diffraction

What happens when a planar wavefront of light interacts with an aperture?

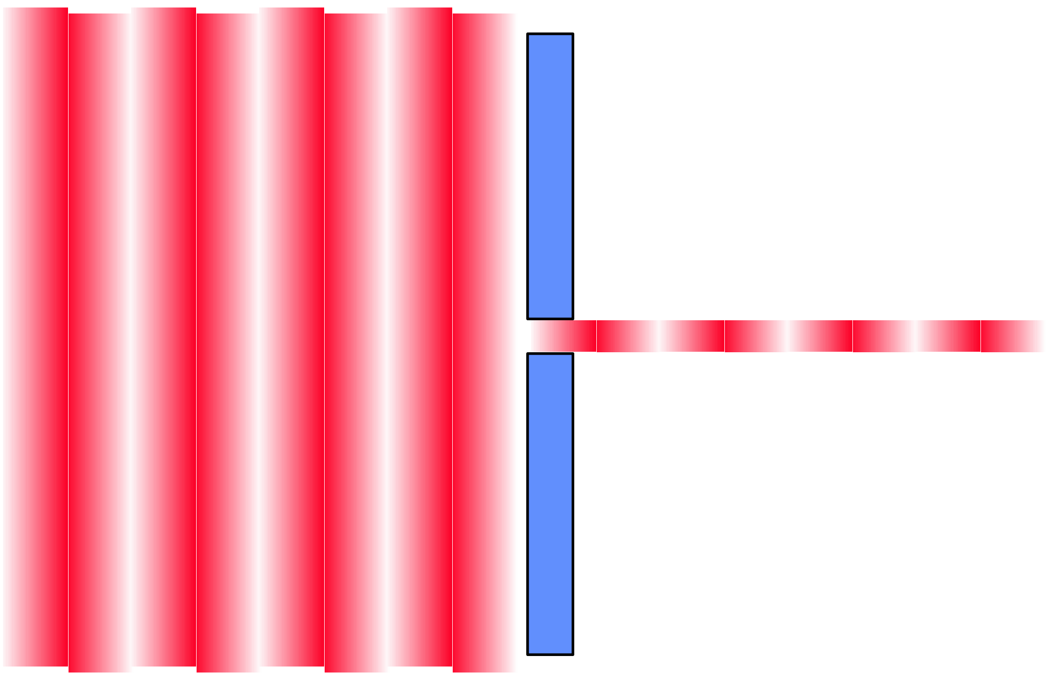
If the aperture is large compared to the wavelength you would expect this....



…Light propagating in a straight path.

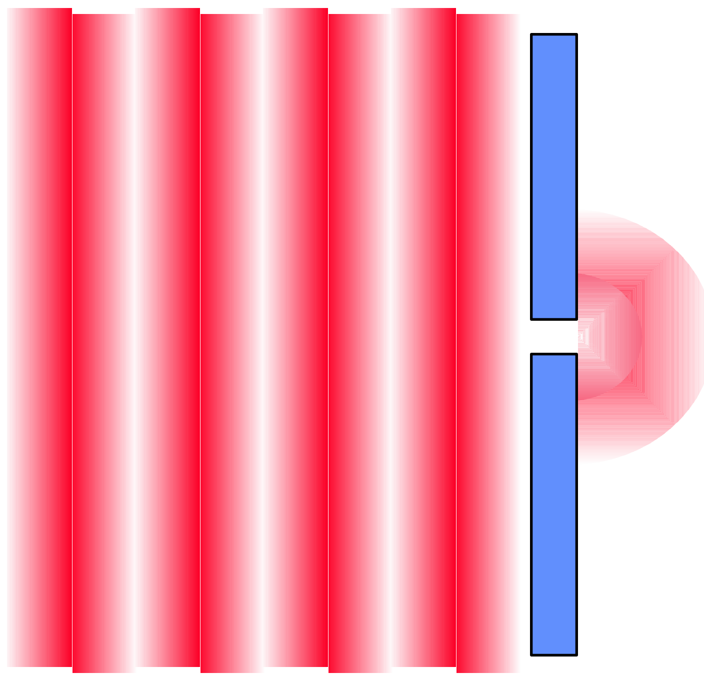
Diffractio
n

If the aperture is small compared to the wavelength would you expect this?



Not really…

Diffraction

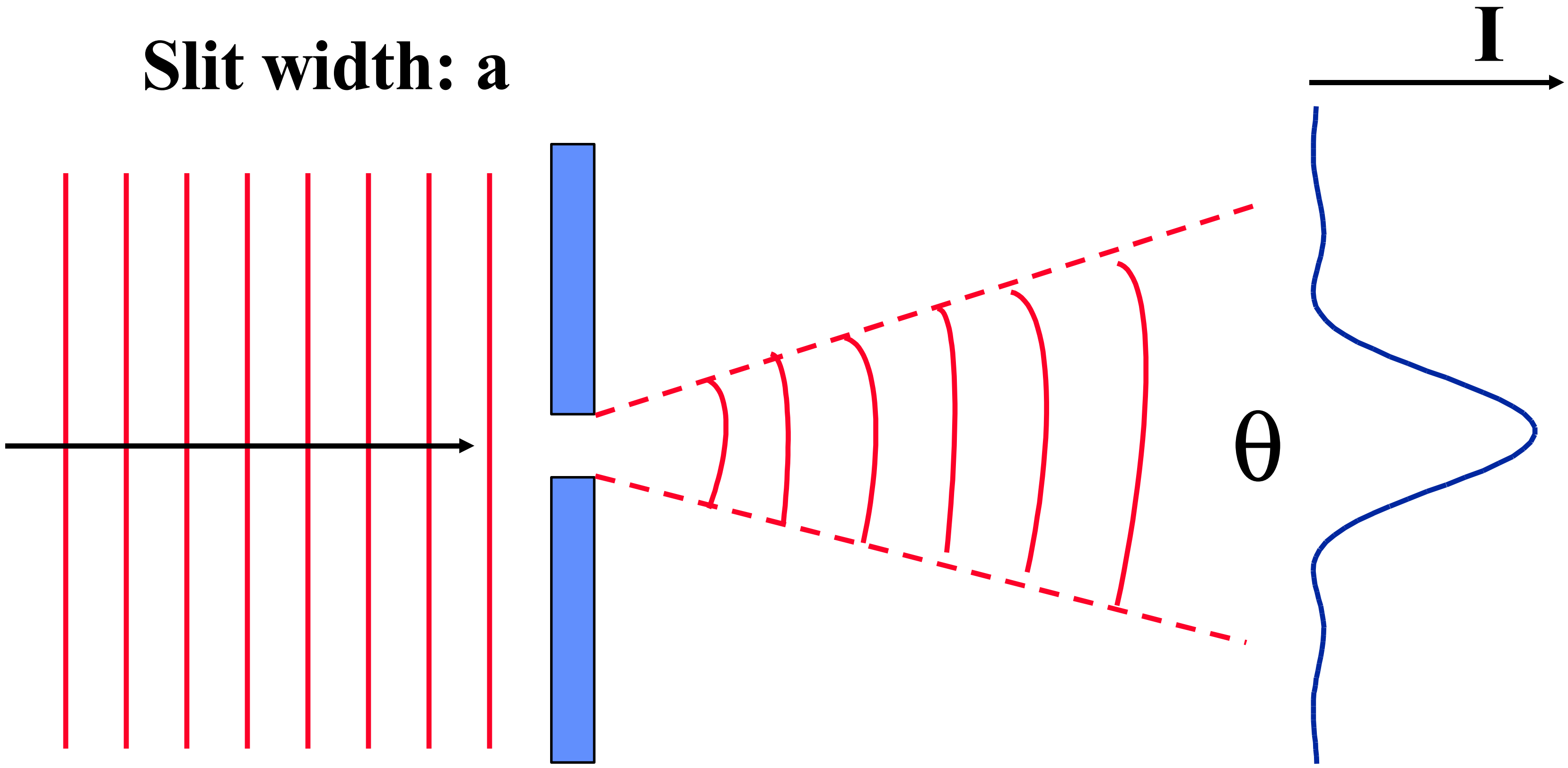


In fact, what happens is that: a spherical wave propagates out from the aperture.

All waves behave this way.

This phenomenon of light spreading out in a broad pattern, instead of following a straight path, is called: DIFFRACTION

Diffraction

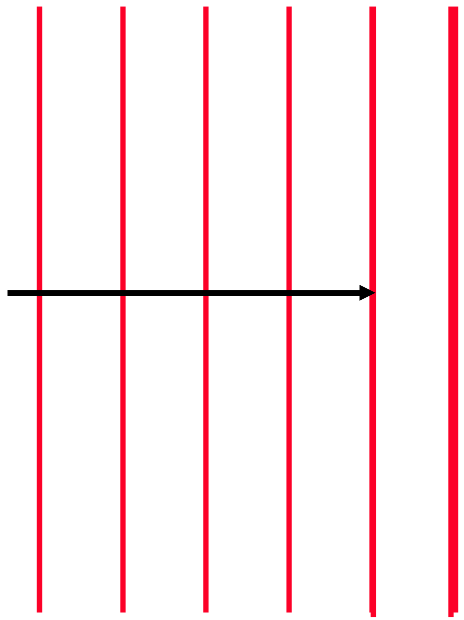


(Actual intensity distribution)

Angular Spread: $\theta \sim \lambda/a$

Huygen's Principle

Huygen first explained this in 1678 by proposing that all planar wavefronts are made up of lots of spherical wavefronts..



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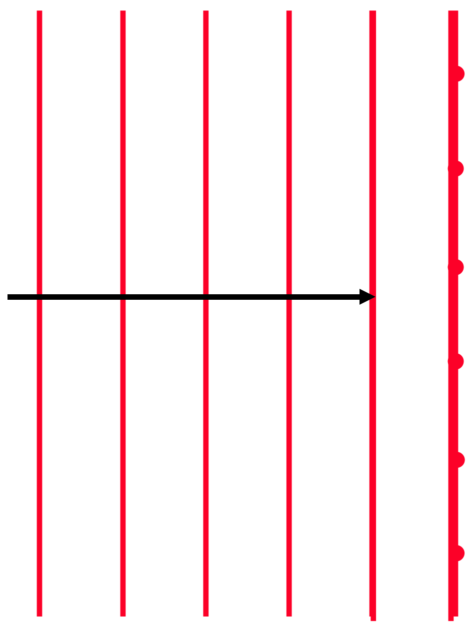

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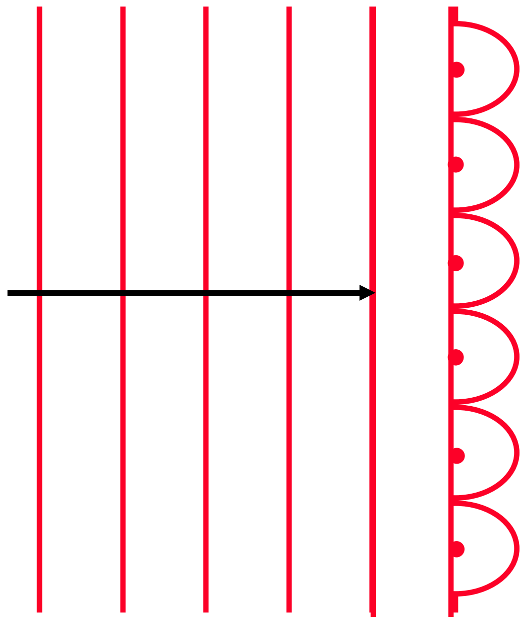

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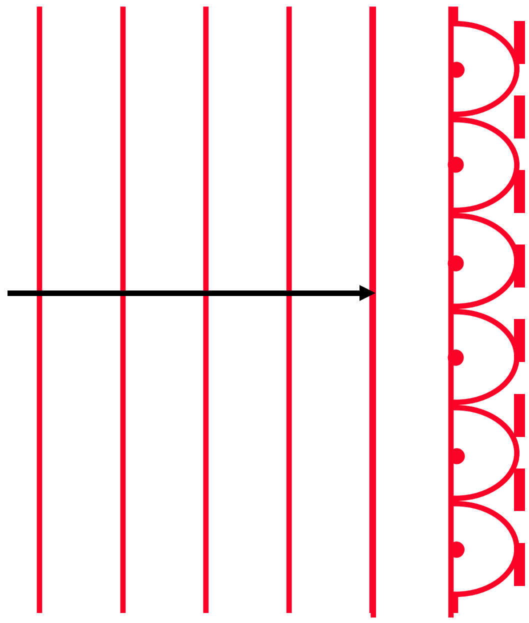

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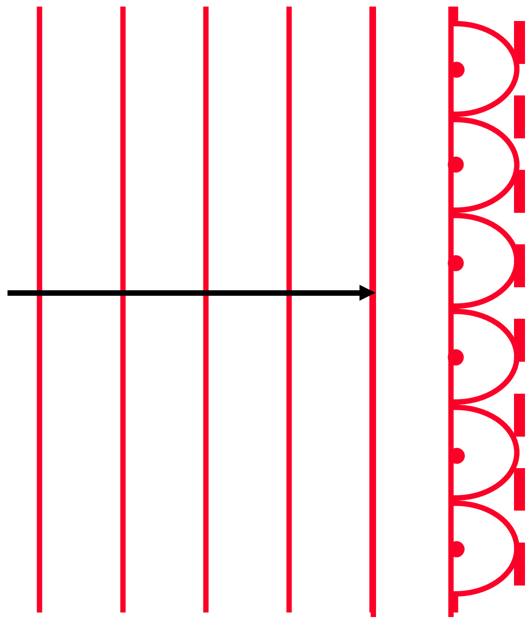
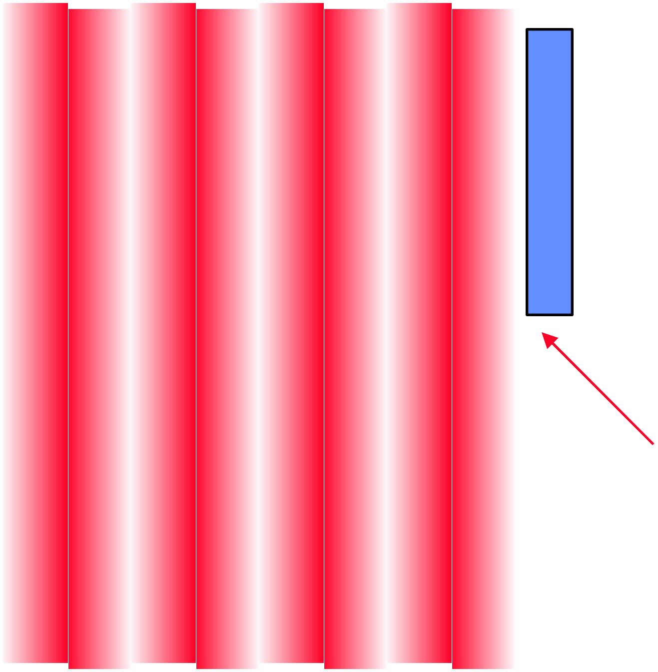


That is, you see how light propagates by breaking a wavefront into little bits, and then draw a spherical wave emanating outward from each little bit. You then can find the leading edge a little later simply by summing all these little “wavelets”

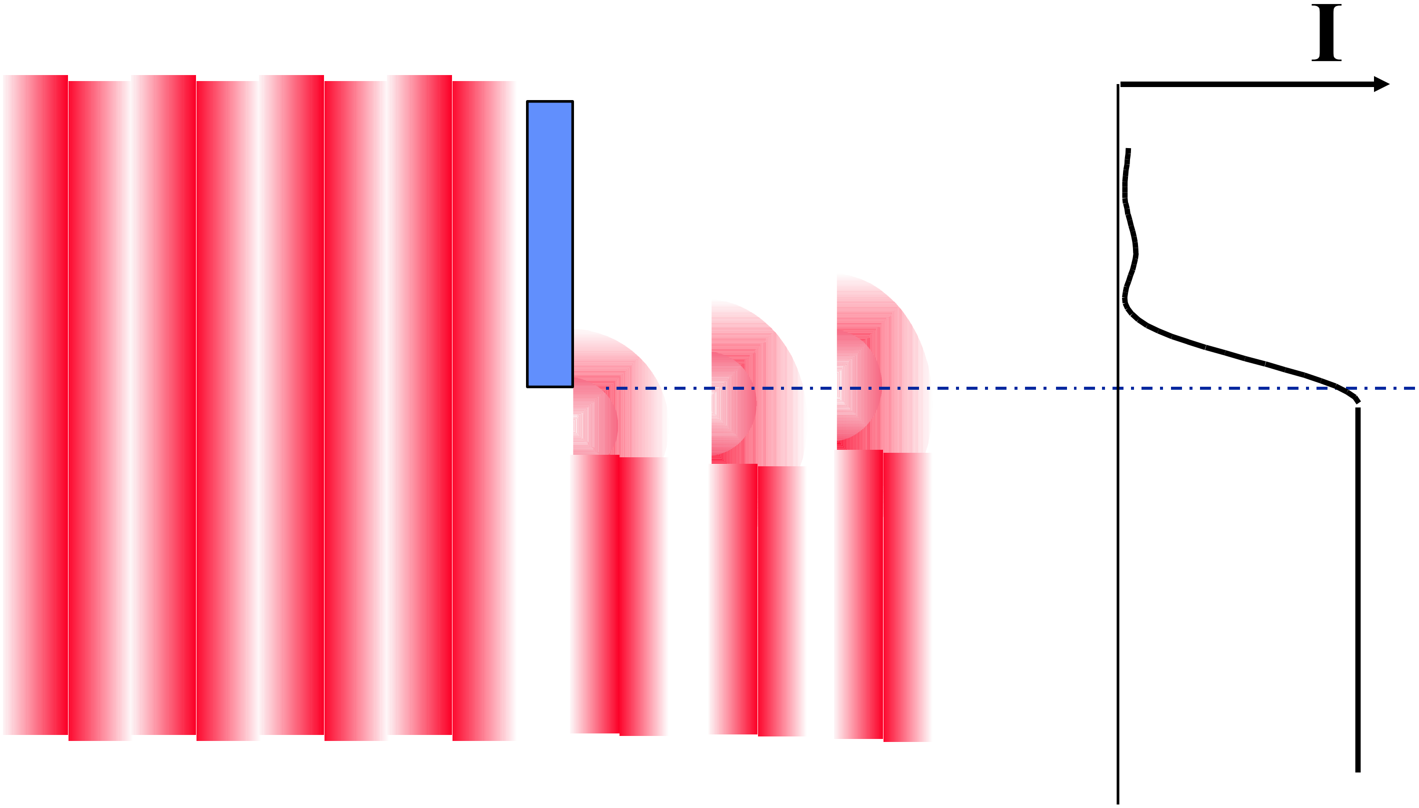
It is possible to explain reflection and refraction this way too.

Diffraction at Edges

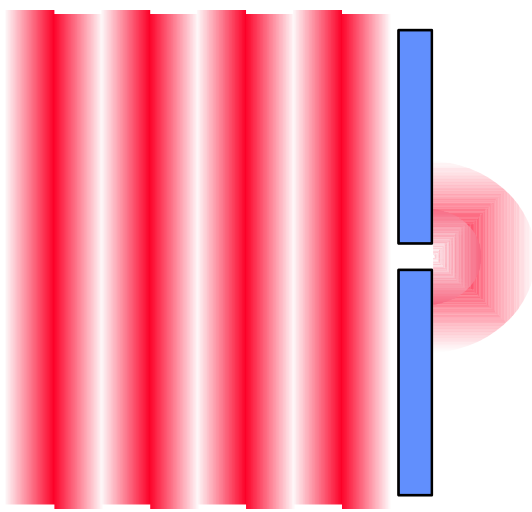


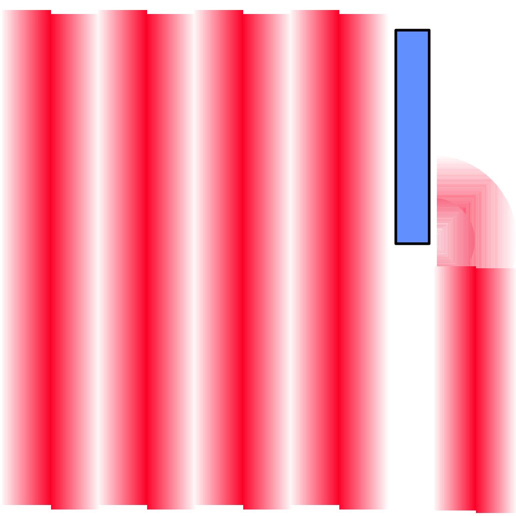
what happens to the shape of the field at this point?

Diffraction at Edges



As The Wave Propagates Out Spherically Its Intensity Decreases

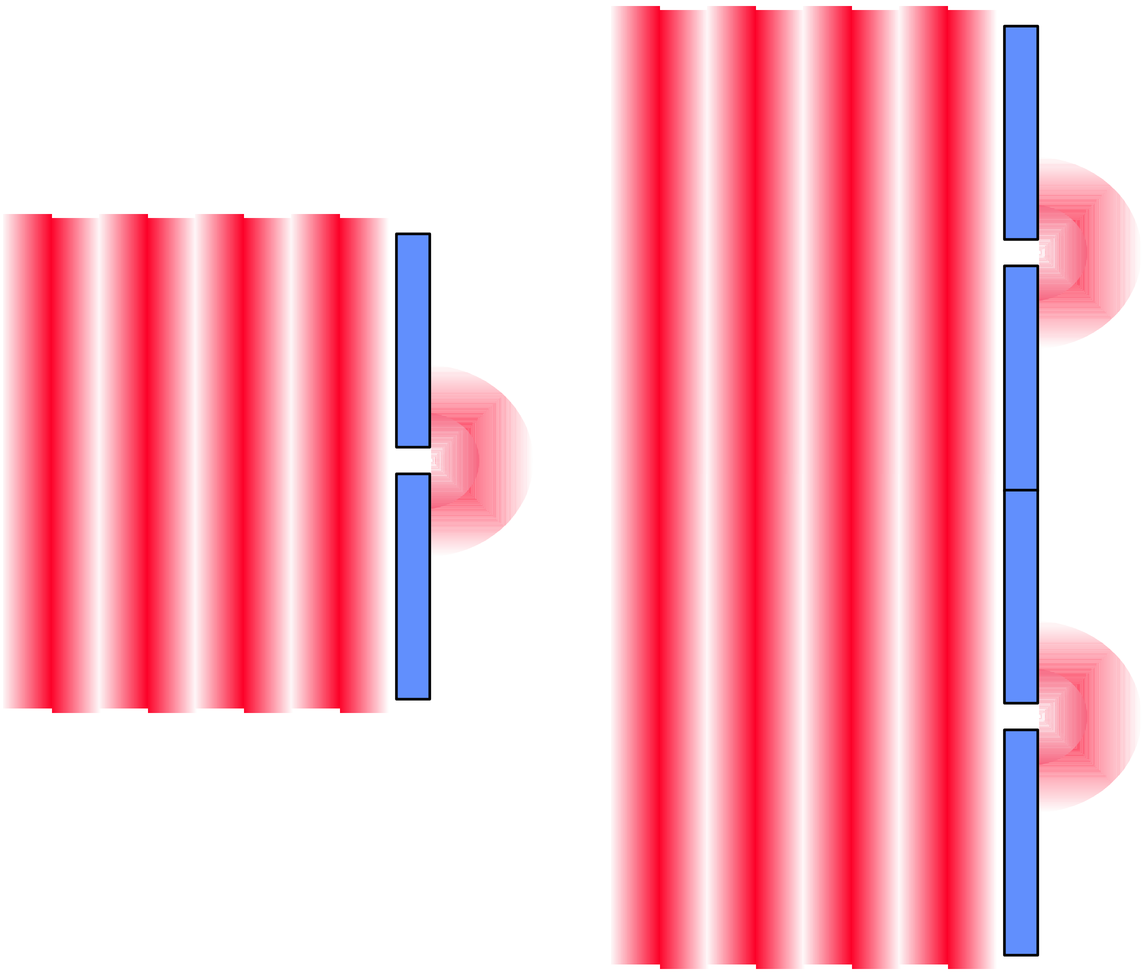




.....this happens with an edge too..

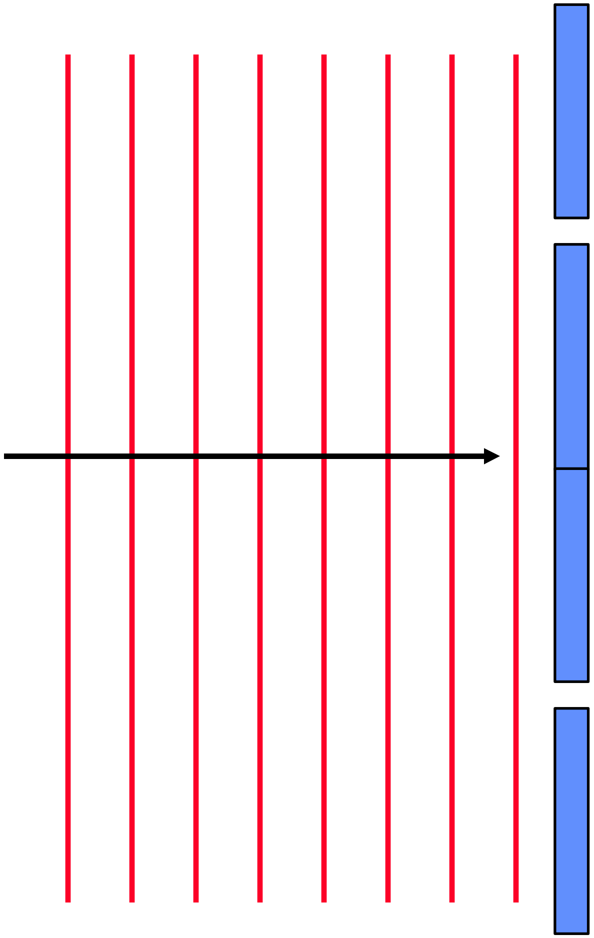
Diffraction places a finite limit on the formation of images

Double-Slit Interference



Because they spread, these waves will eventually interfere with one another and produce interference fringes

Double-Slit Interference



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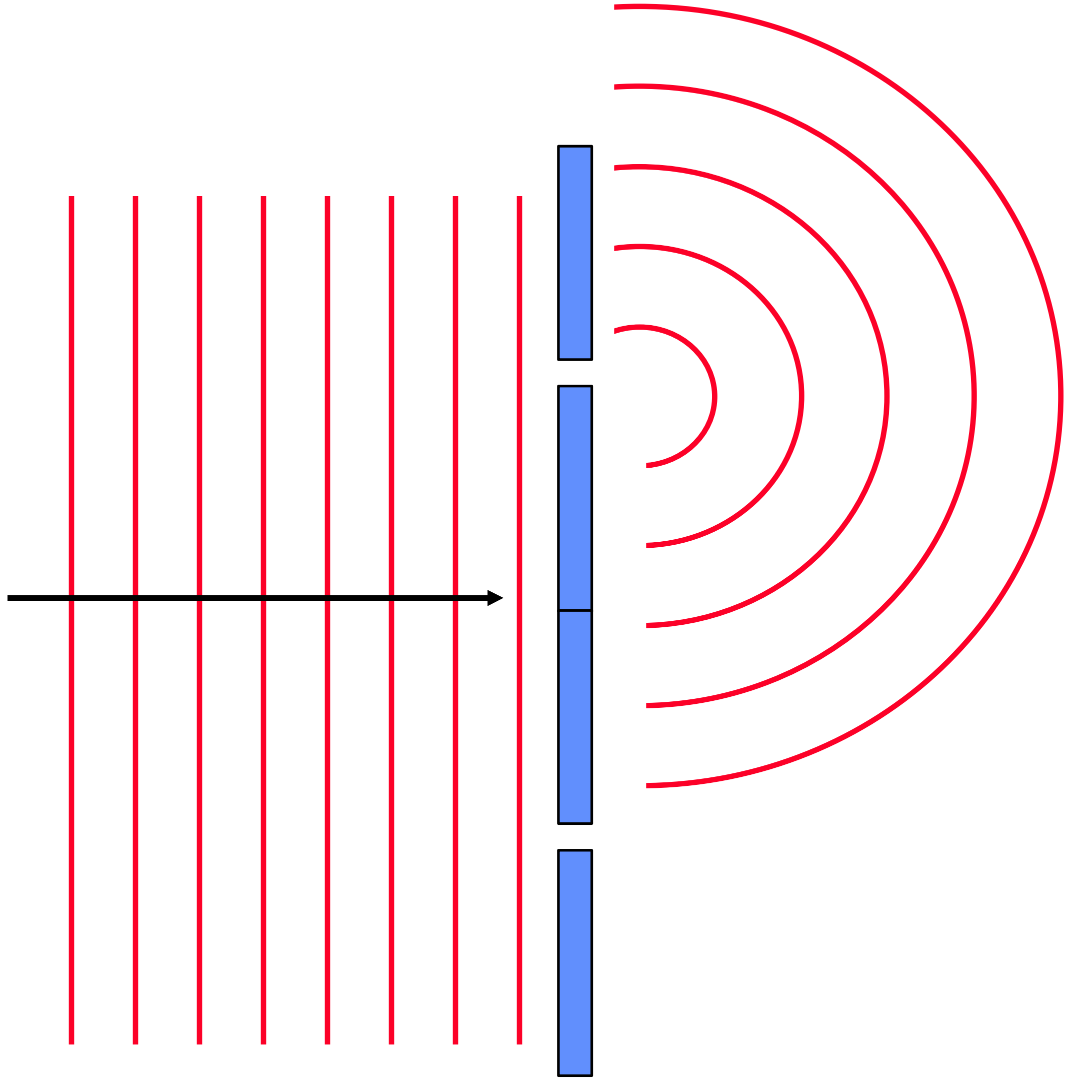

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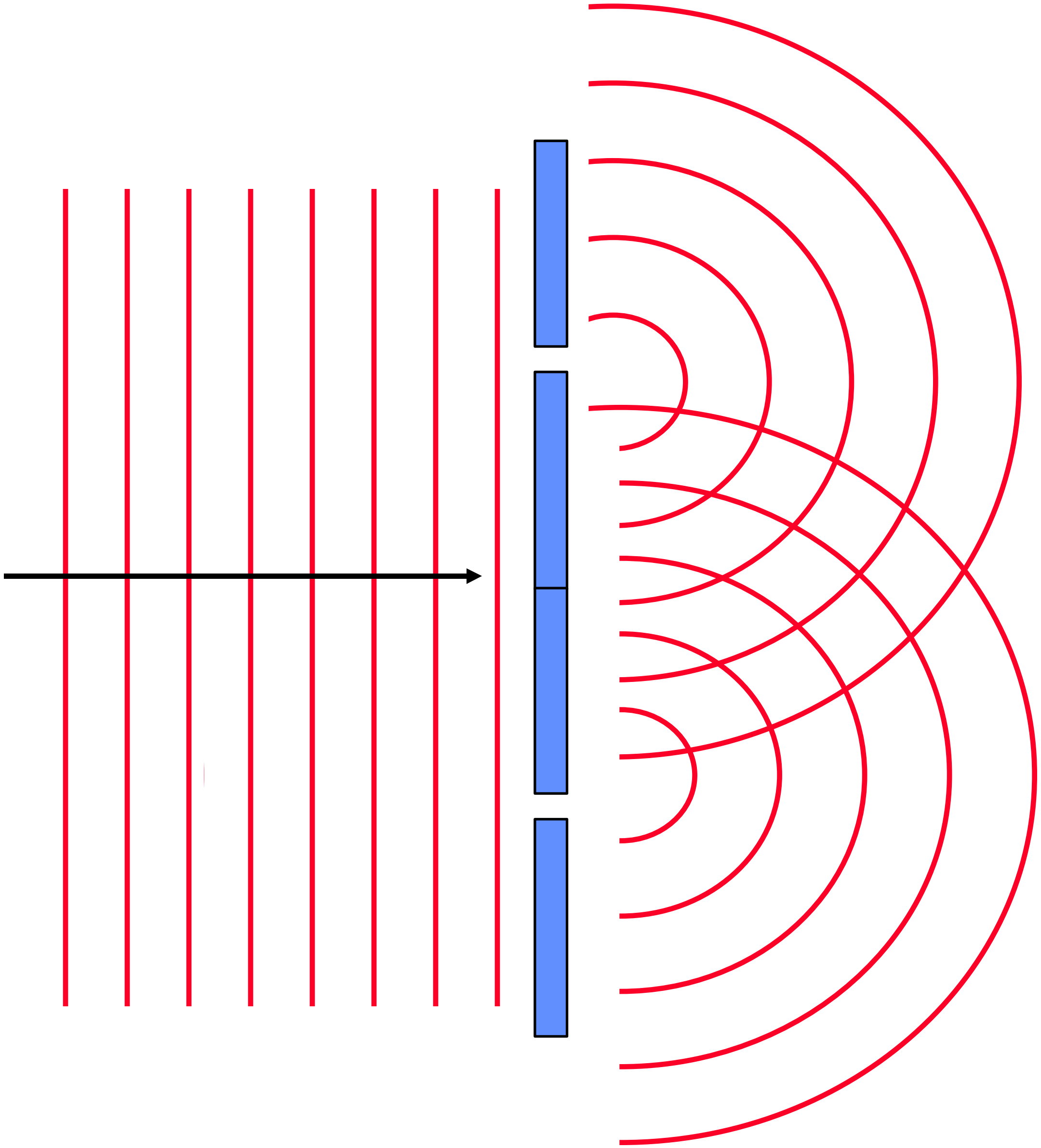

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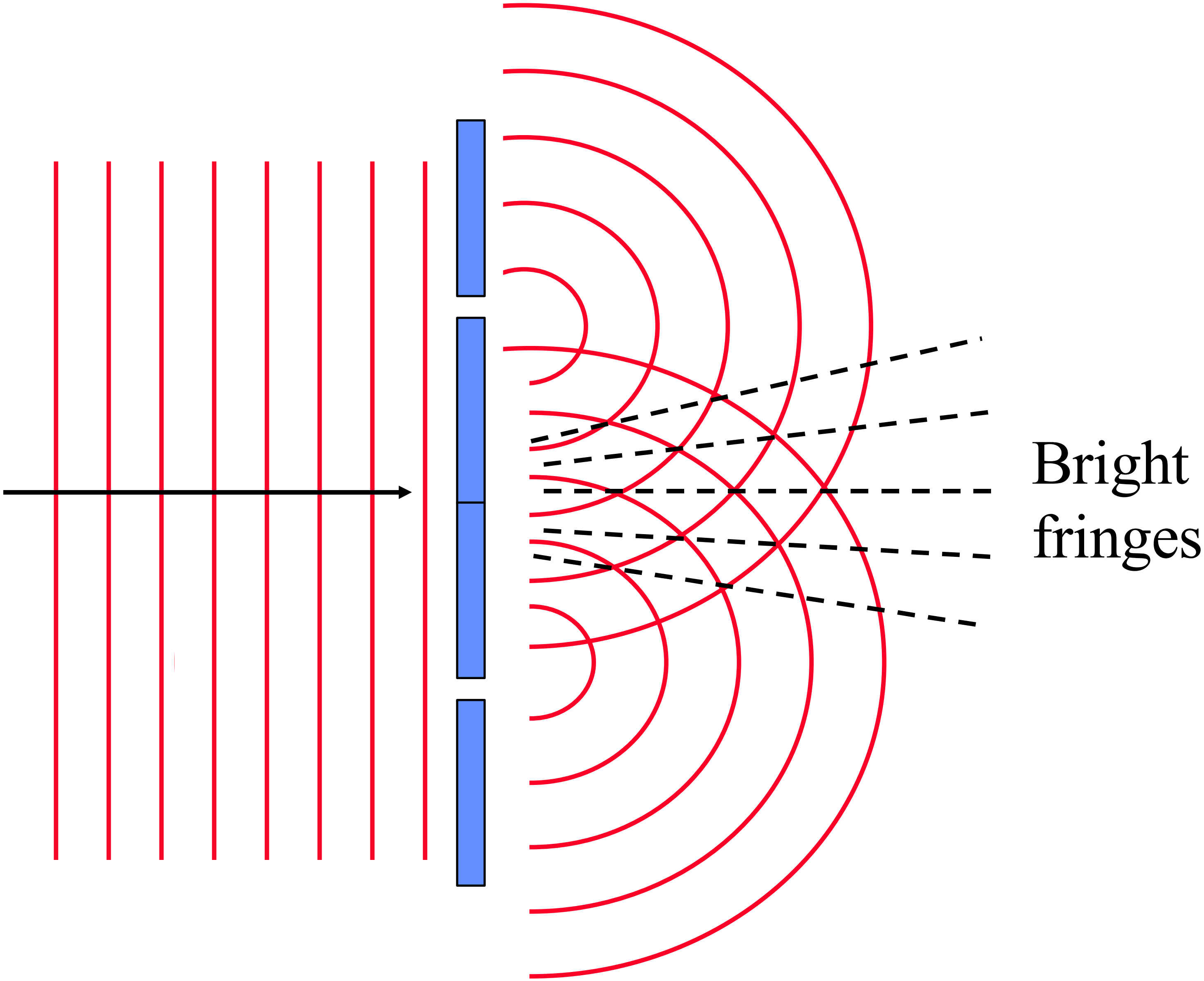

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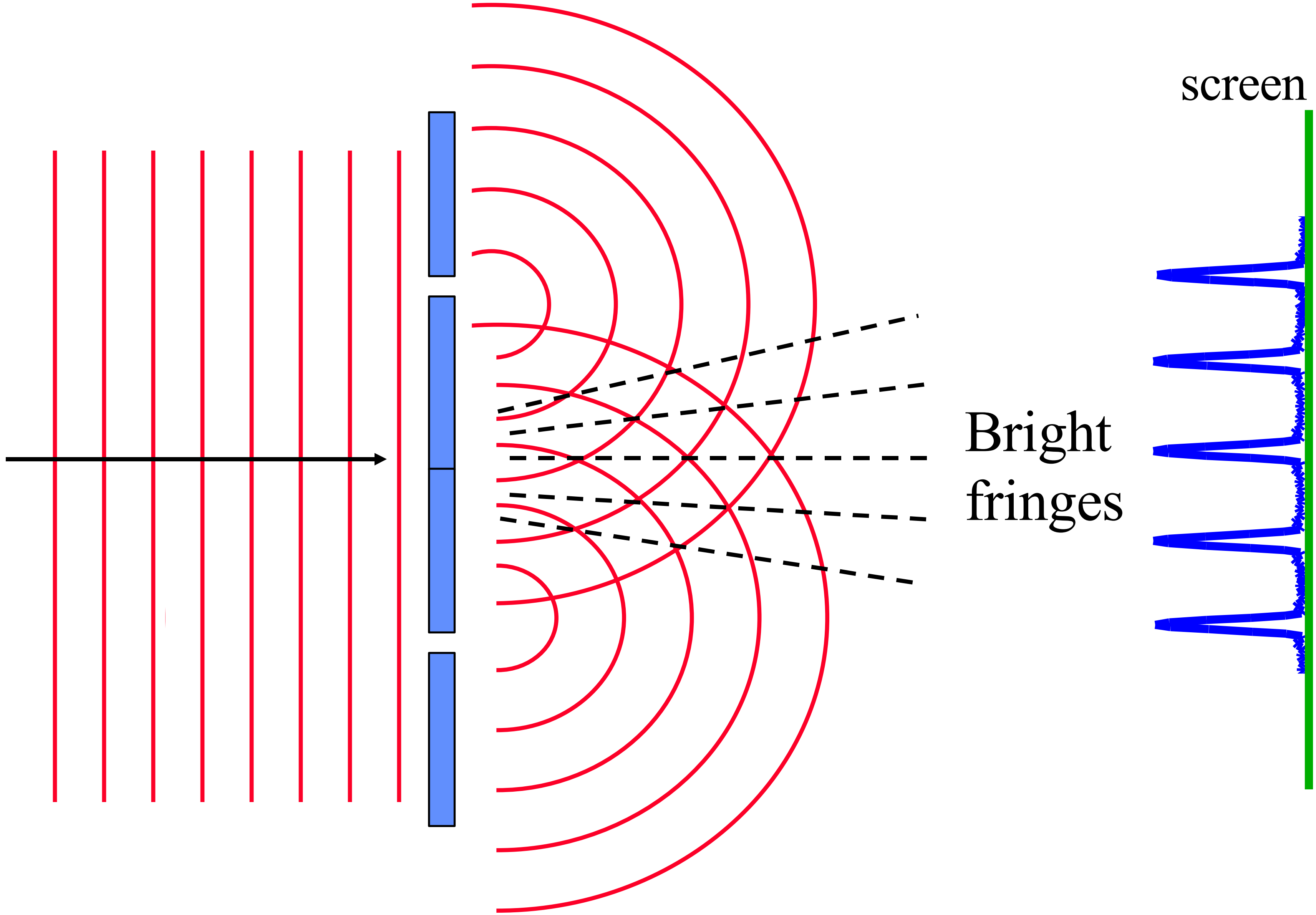

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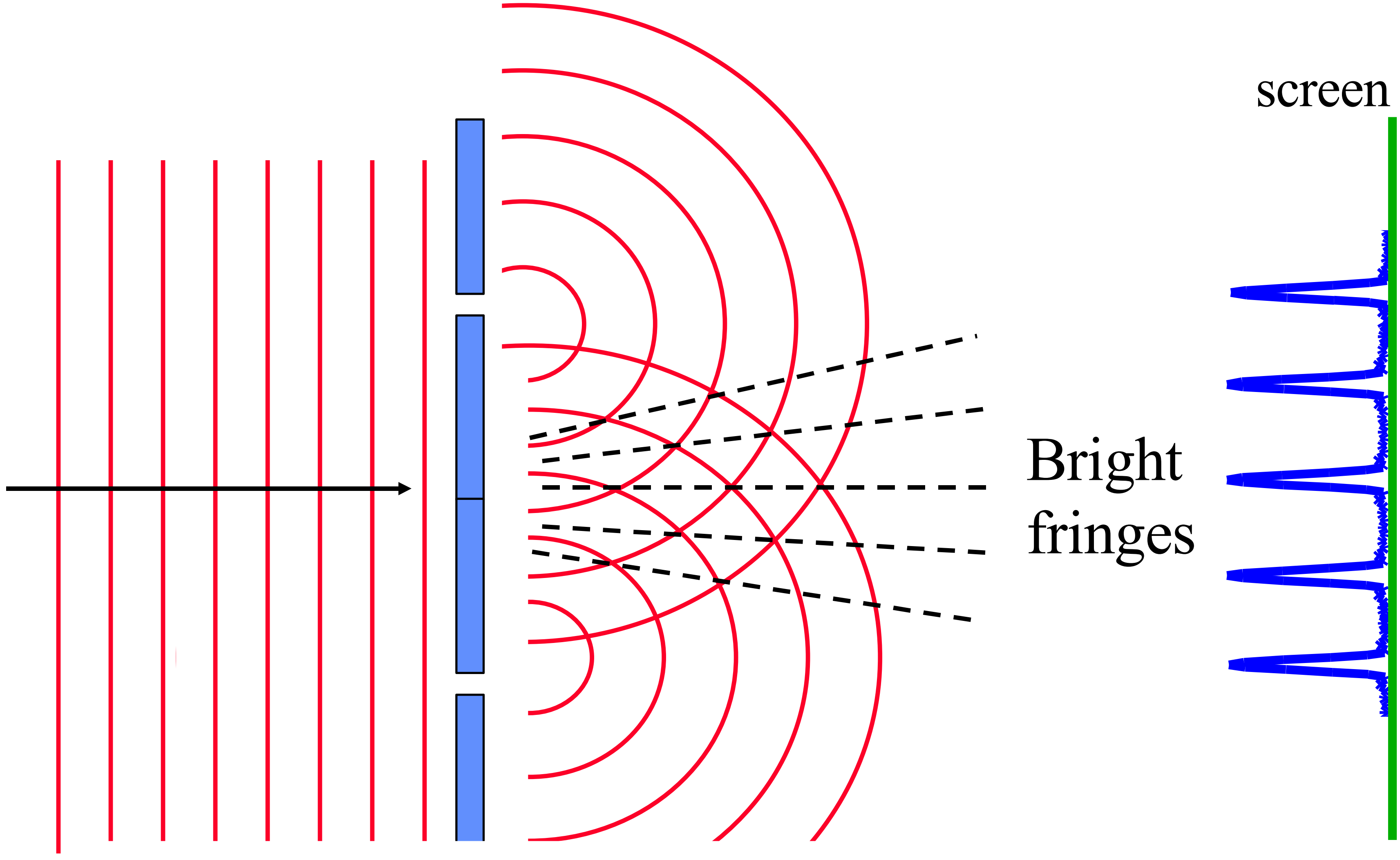
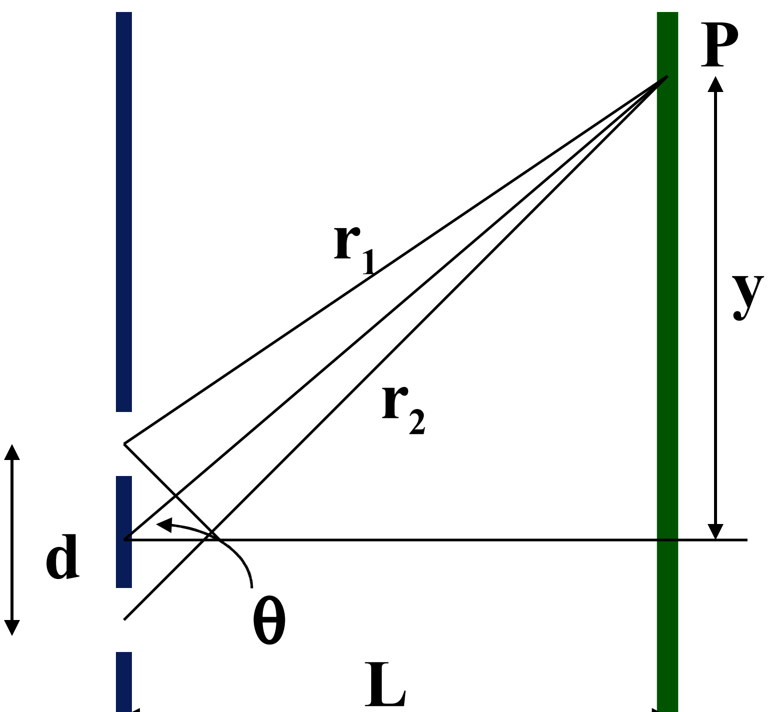


Thomas Young (1802) used double-slit interference to prove the wave nature of light.

Double-Slit Interference

Light from the two slits travels different distances to the screen. The difference $r_1 - r_2$ is very nearly $d \sin\theta$. When the path difference is a multiple of the wavelength these add constructively, and when it's a half-multiple they cancel.



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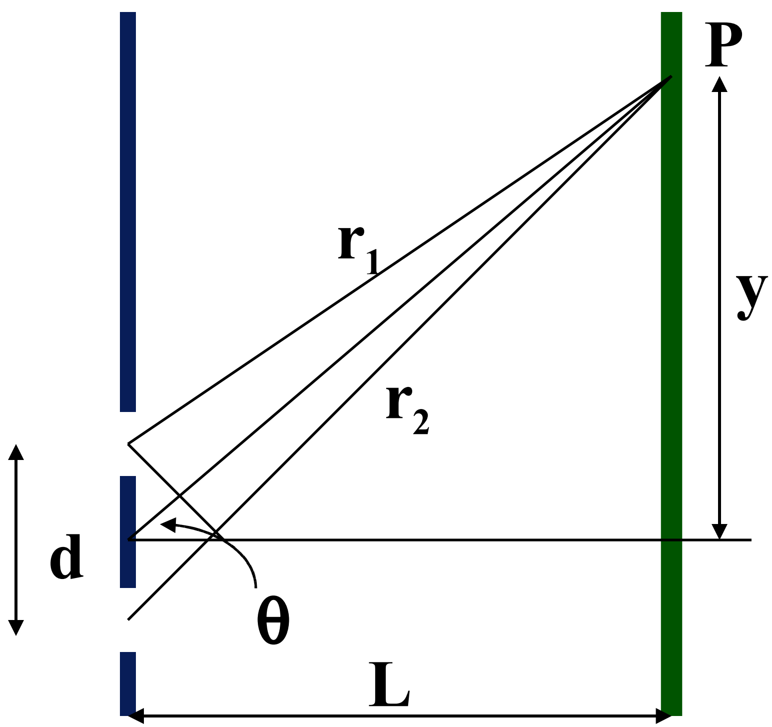

$d \sin\theta = m\lambda$ bright fringes

$d \sin\theta = (m+1/2)\lambda$ dark fringes

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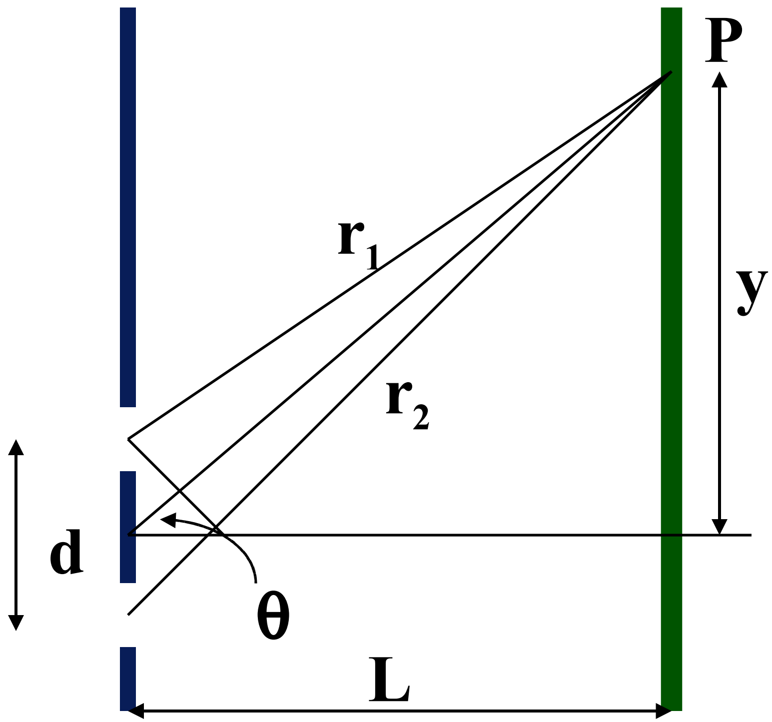


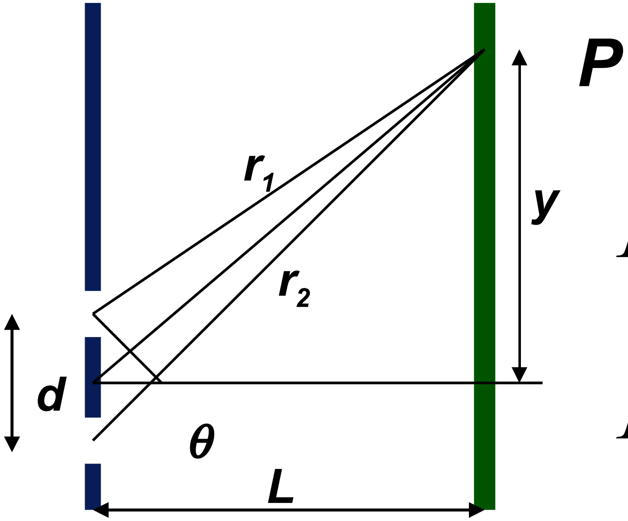
$d \sin\theta = m\lambda$ bright fringes

$d \sin\theta = (m+1/2)\lambda$ dark fringes

Now use $y = L \tan\theta$; for small y:

$y_{bright} = m\lambda L/d$

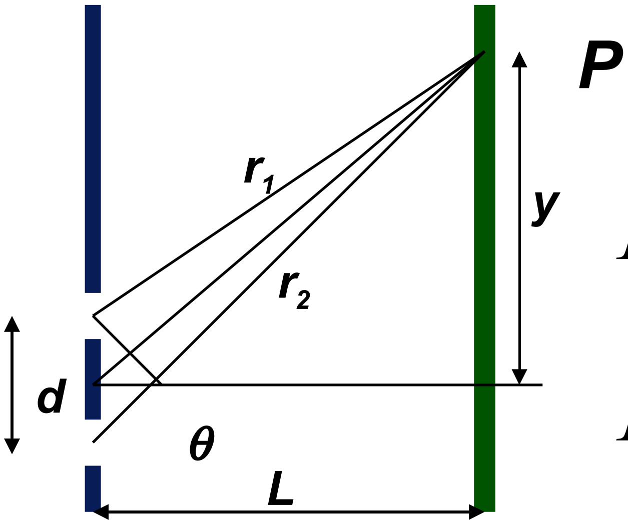
$y_{dark} = (m+1/2)\lambda L/d$



What is the field amplitude?

$$E = E_1 + E_2 = E_P[\sin\omega t + \sin(\omega t + \phi)]$$

$$E = 2E_P \sin(\omega t + \frac{\phi}{2})\cos(\frac{\phi}{2})$$



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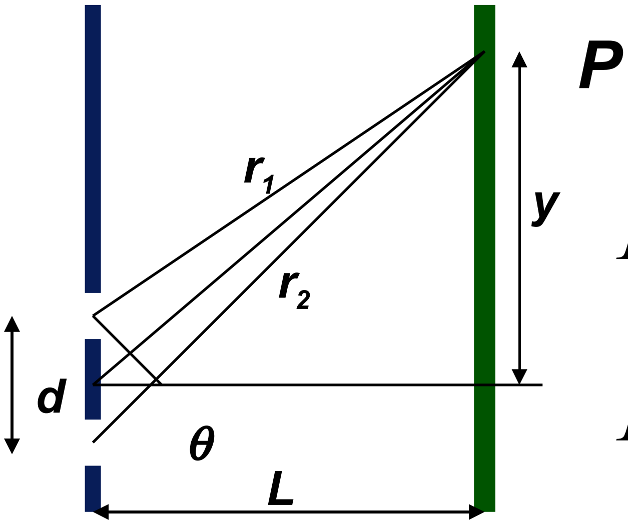
Phase is $k(r_1 - r_2)$:

$$\phi = 2\pi \frac{d\sin\theta}{\lambda}$$



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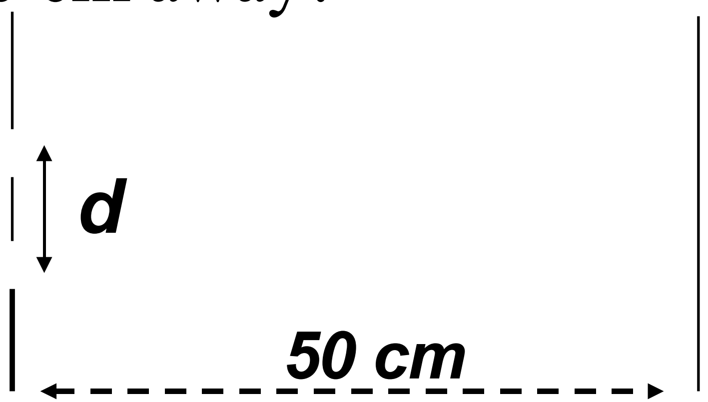
$$\therefore\ E = 2E_P \sin(\omega t + \frac{\phi}{2})\cos(\frac{\pi d \sin\theta}{\lambda})$$

$$\overline{S} = \frac{[2E_P \cos(\pi d \sin\theta / \lambda)]^2}{2\mu_0 c} = 4\overline{S_0}\cos^2(\frac{\pi d \sin\theta}{\lambda})$$

$$\overline{S} = 4\overline{S_0}\cos^2(\frac{\pi d}{\lambda L} y)$$

Example: Double Slit Interference

Light of wavelength $\lambda = 500$ nm is incident on a double slit spaced by $d = 50$ μm. What is the fringe spacing on the screen, 50 cm away?



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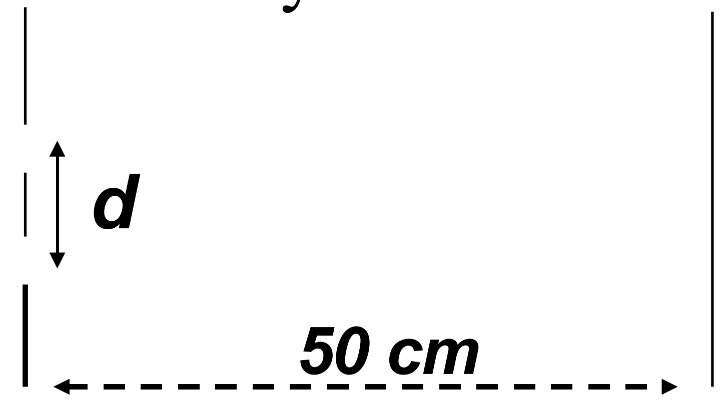


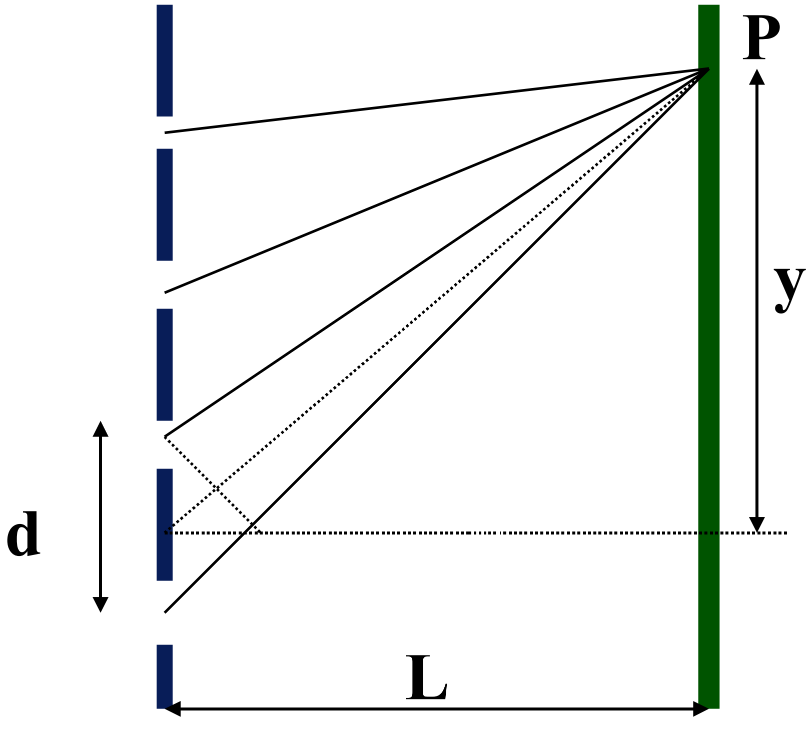
$$\Delta y = L\lambda / d$$

$$= \left(50\times 10^{-2} m\right)\left(500\times 10^{-9} m\right) / \left(50\times 10^{-6} m\right)$$

$$= 5 mm$$

Multiple Slit Interference

With more than two slits, things get a little more complicated



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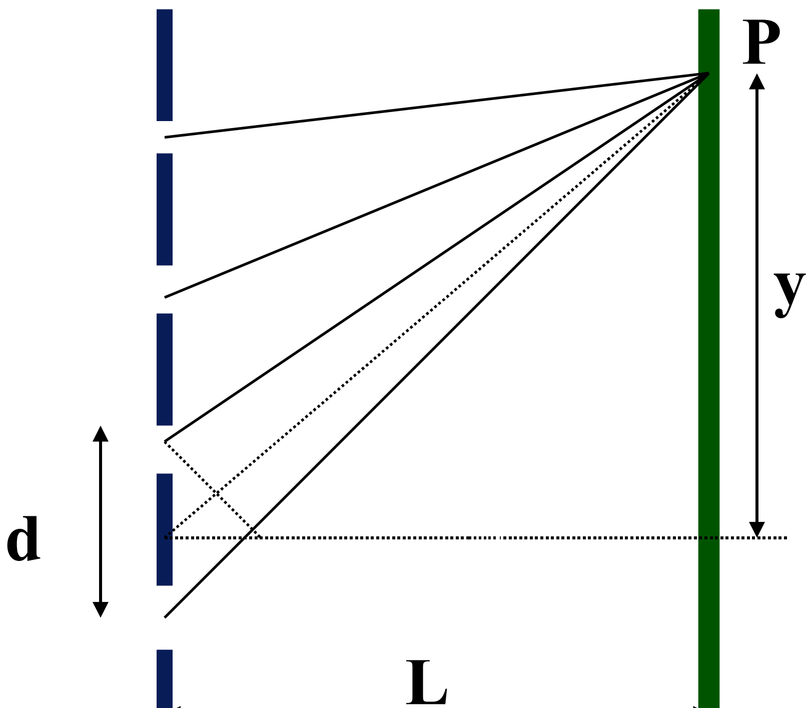

Now to get a bright fringe, many paths must all be in phase.

The brightest fringes become narrower but brighter;

and extra lines show up between them.

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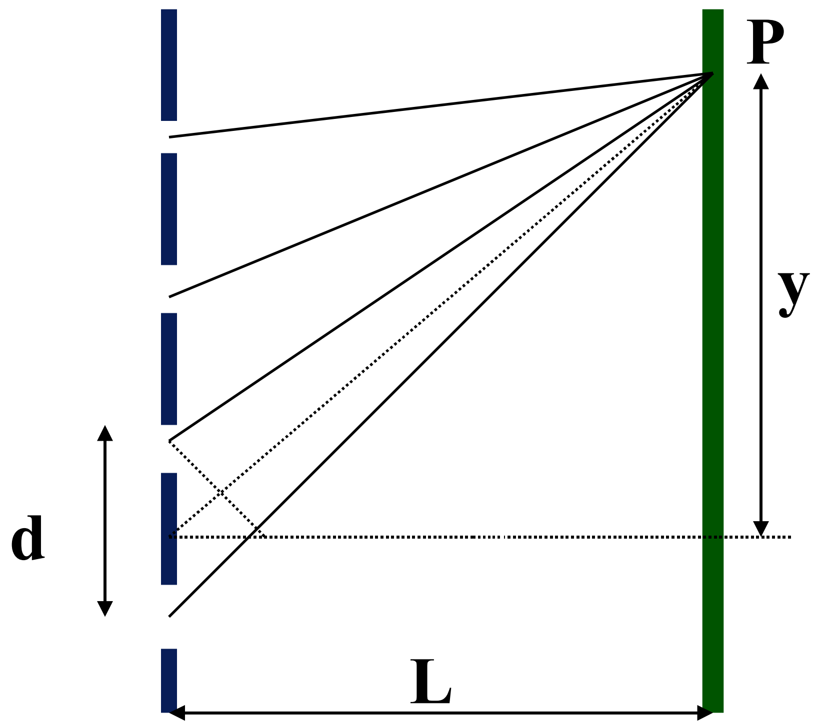

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Such an array of slits is called a "<u>Diffraction Grating</u>"

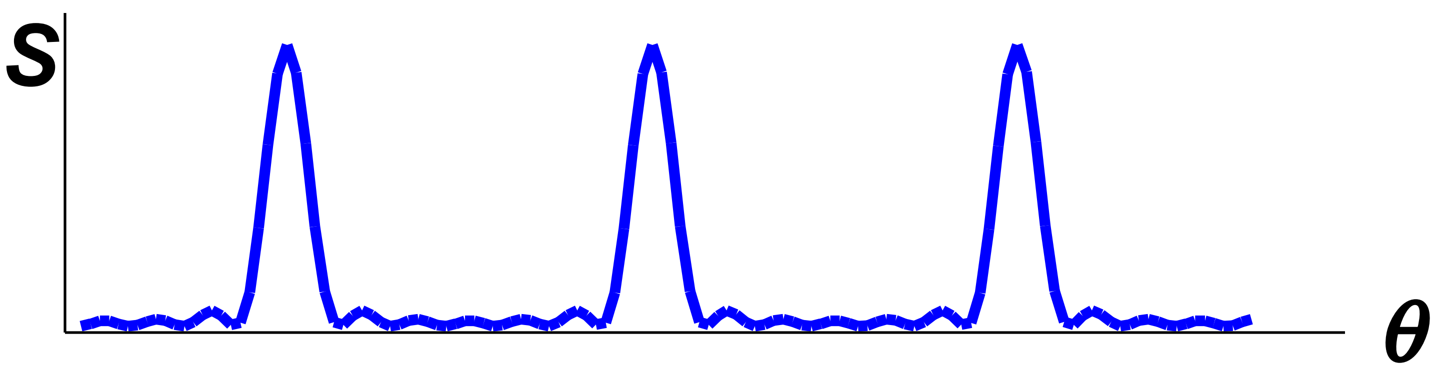
Multiple Slit Interference

All of the lines show up at the set of angles given by:

$$d\sin\theta = (m/N)\lambda$$

(N = number of slits). Most of these are not too bright. The very bright ones are for m a multiple of N.

We won't worry about the math here, just look at the general form:



Single Slit Diffraction

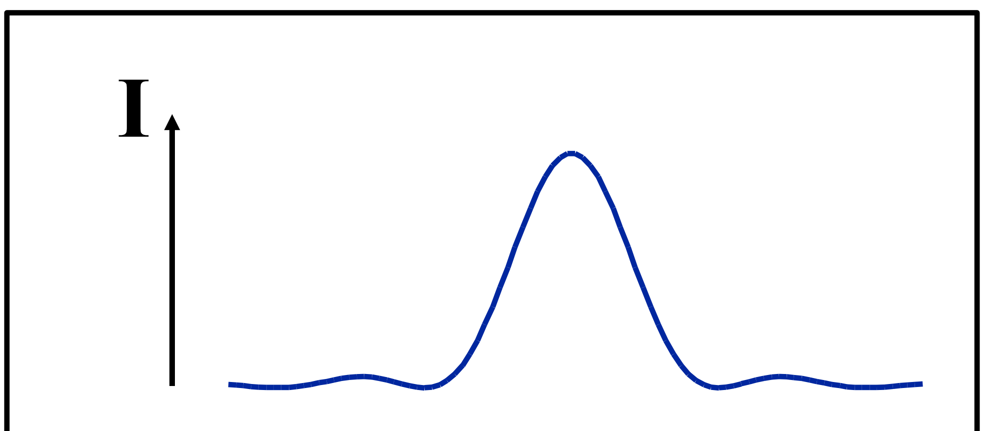
Each point in the slit acts as a source of spherical wavelets

For a particular direction θ, wavelets will interfere, either constructively or destructively.

Result:

$$\overline{S_\theta} = \overline{S_o}\left[\frac{\sin(\frac{\pi a}{\lambda}\sin\theta)}{\frac{\pi a}{\lambda}\sin\theta}\right]^2$$

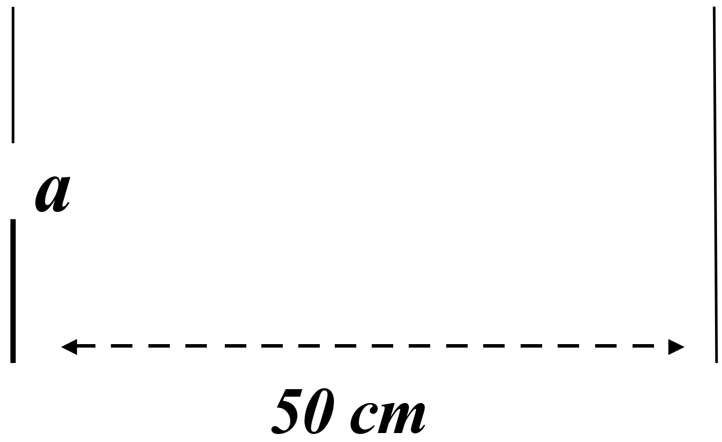
This is for a slit of width a.



This gives the angular spread ~ λ/a.

Example: Single Slit Diffraction

Light of wavelength λ = 500 nm is incident on a slit a=50 μm wide. How wide is the intensity distribution on the screen, 50 cm away?



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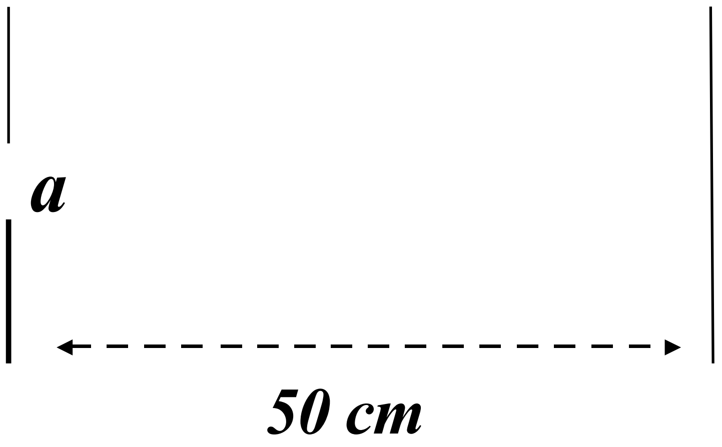

$$\Delta\theta \approx \lambda / a$$

$$\Delta y \approx L\,\Delta\theta = \left(50\times 10^{-2}\,m\right)\left(500\times 10^{-9}\,m\right)/\left(50\times 10^{-6}\,m\right) = 5\,mm$$

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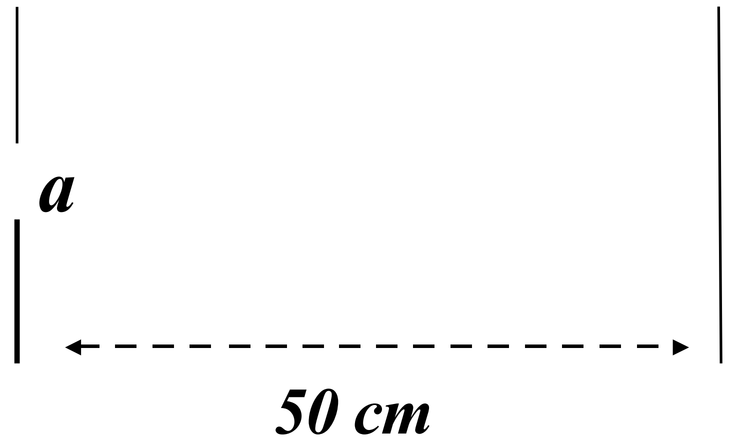

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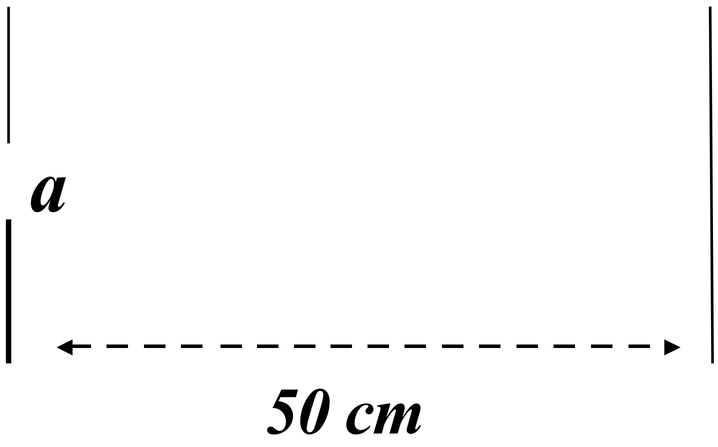

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What happens if the slit width is doubled?

The spread gets cut in half.

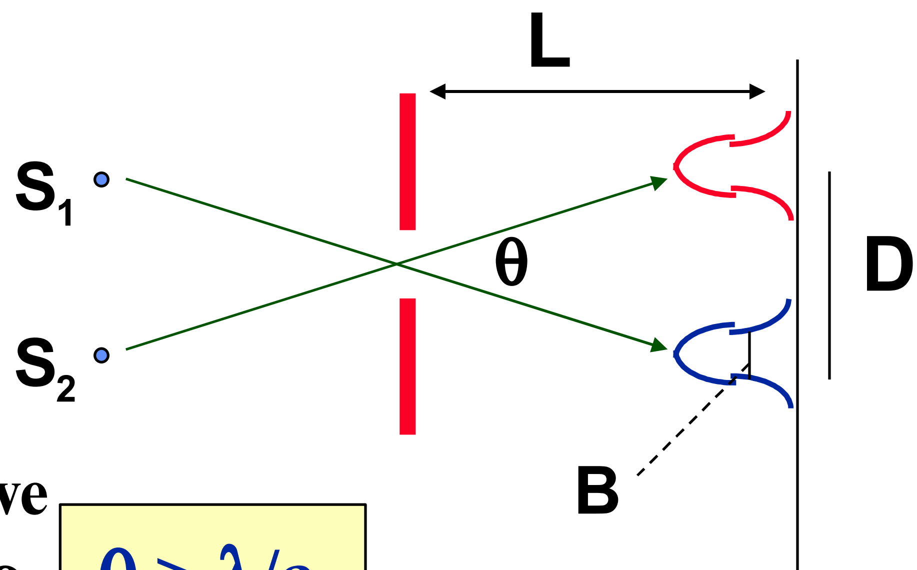
The Diffraction Limit

Diffraction therefore imposes a fundamental limit on the resolution of optical systems:

Suppose we want to image 2 distant points, S_1 and S_2 through an aperture of width a:

The image separation is $D \sim L\sin\theta \sim L\theta$.

The image blur is $B \sim L\lambda/a$



To resolve individual points, we want: **separation > blur** so $\theta > \lambda/a$