Chapter 34

(continued)

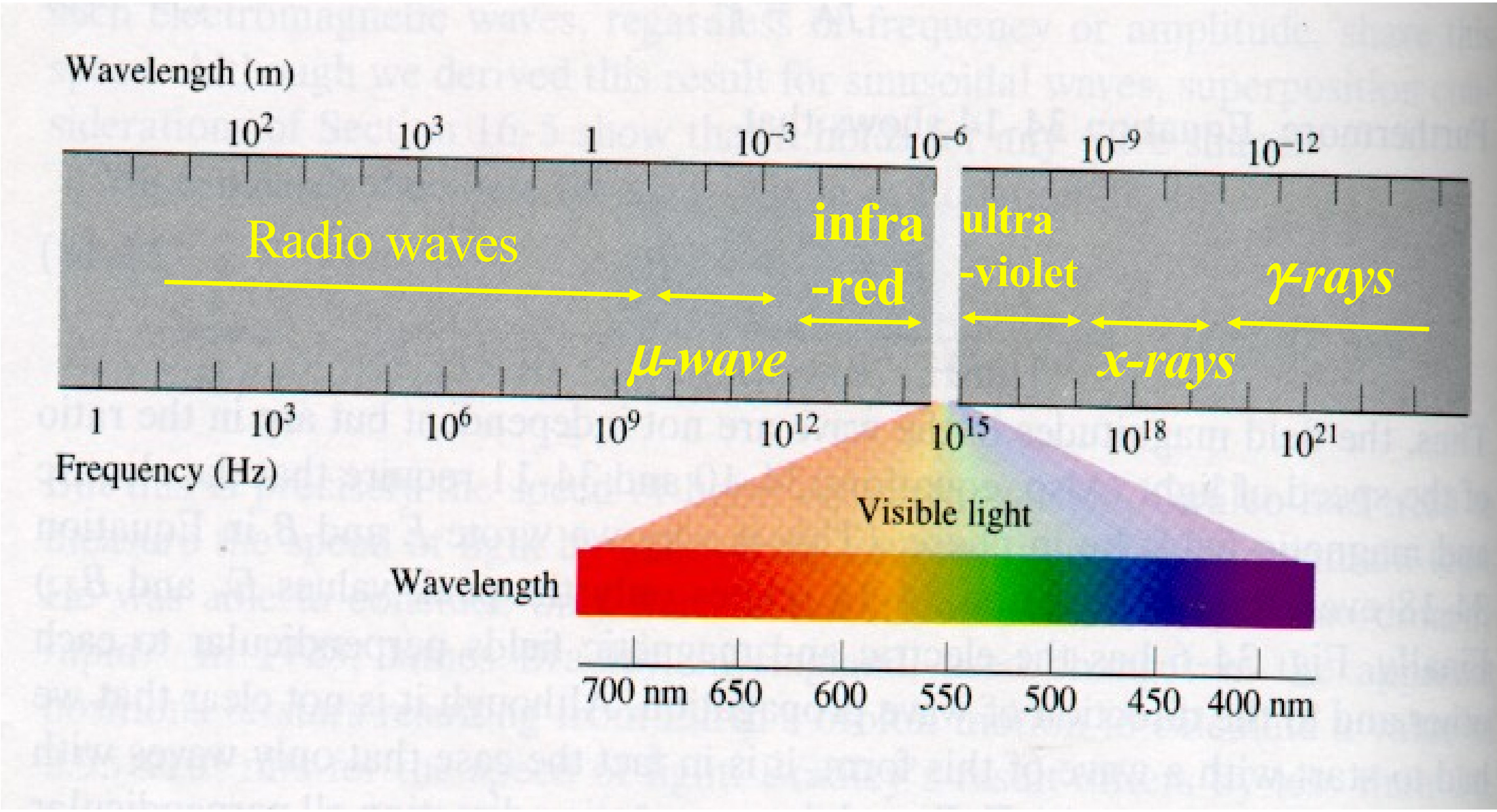
The Laws of Electromagnetism

Maxwell's Equations

Displacement Current

Electromagnetic Radiation

The Electromagnetic Spectrum



Maxwell's Equations of Electromagnetism in Vacuum (no charges, no masses)

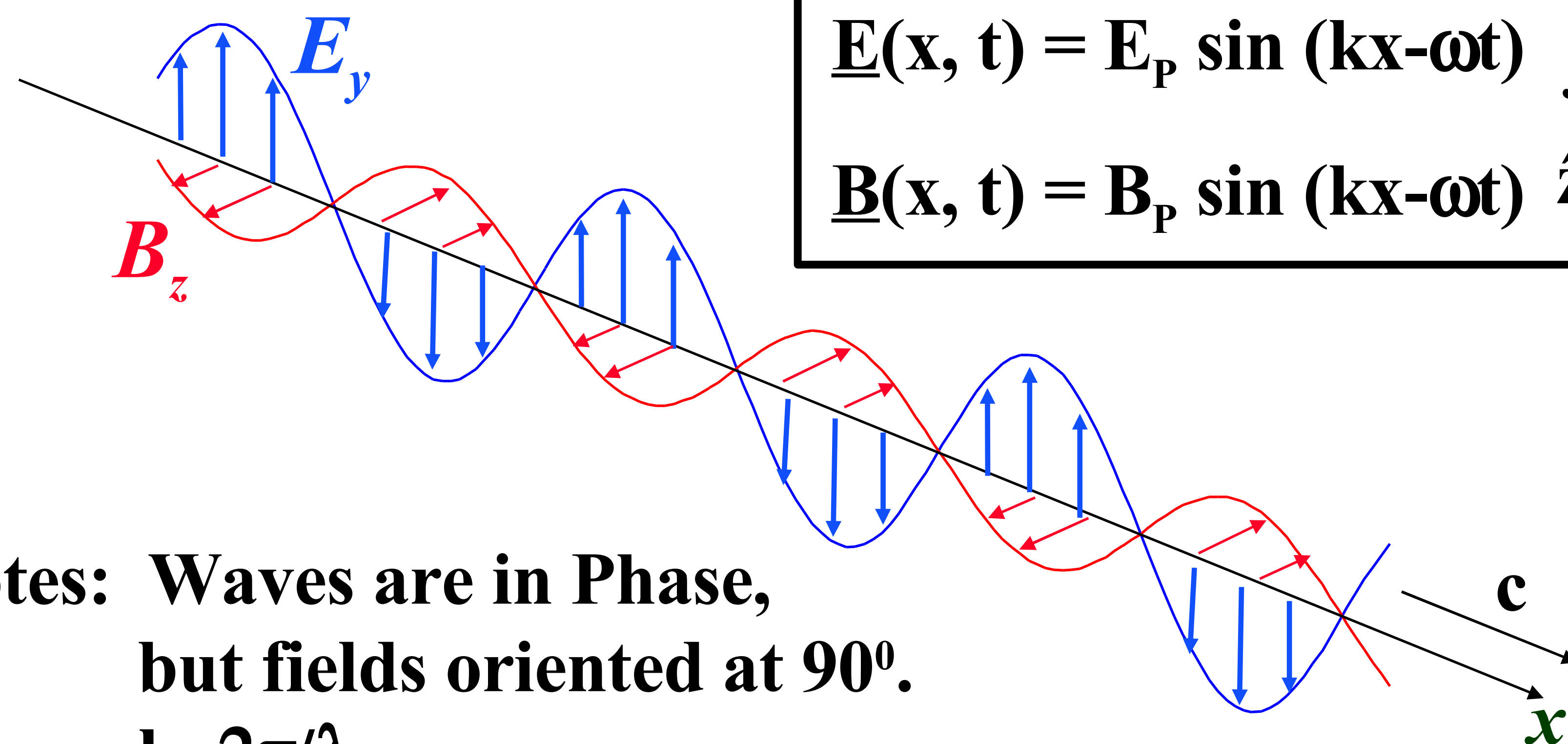
$$\oint \underline{E} \bullet \underline{dA} = 0$$

$$\oint \underline{B} \bullet \underline{dA} = 0$$

$$\oint \underline{E} \bullet \underline{dl} = -\frac{d\Phi_B}{dt}$$

$$\oint \underline{B} \bullet \underline{dl} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Plane Electromagnetic Waves



$\underline{E}(x, t) = E_P \sin(kx - \omega t)\ \hat{j}$

$\underline{B}(x, t) = B_P \sin(kx - \omega t)\ \hat{z}$

Notes: Waves are in Phase,
but fields oriented at 90^0.
$k = 2\pi/\lambda$.
Speed of wave is $c = \omega/k$ ($= f\lambda$)

$$c = 1/\sqrt{\varepsilon_0 \mu_0} = 3 \times 10^8\, m/s$$

At all times E=cB.

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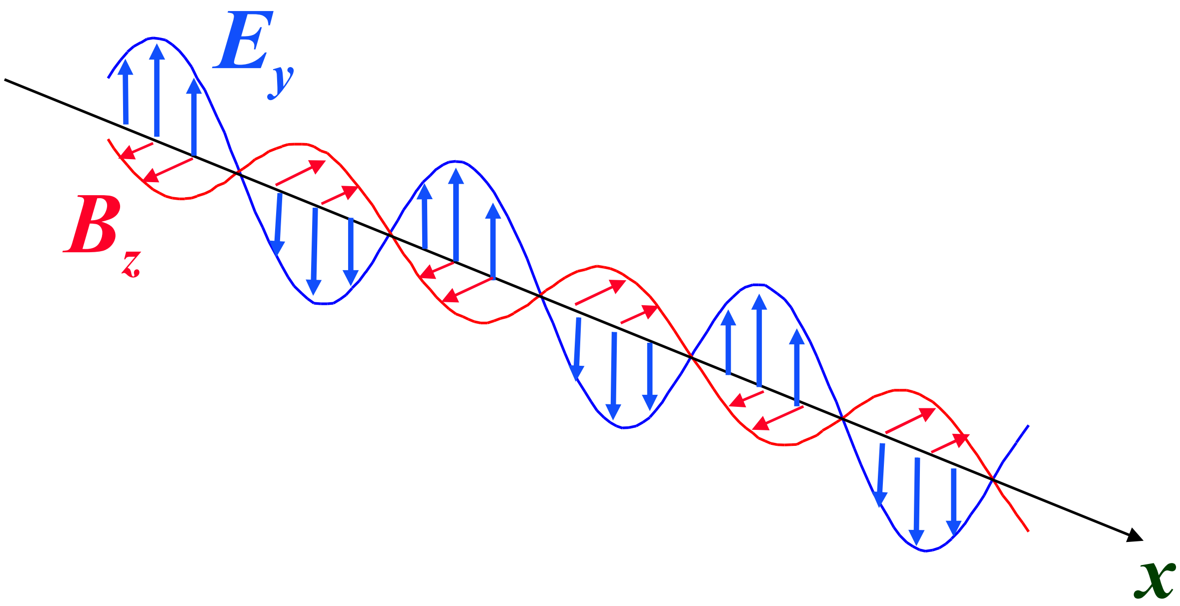


Note: sin(ωt-kx) = -sin(kx-ωt) ⇒ notations are interchangeable.
sin(ωt-kx) and sin(kx-ωt) represent waves traveling <u>towards +x</u>, while sin(ωt+kx) travels <u>towards -x</u>.

Energy in Electromagnetic Waves

- Electric and magnetic fields contain energy, potential energy stored in the field: u_E and u_B

u_E: $\frac{1}{2}\varepsilon_0 E^2$ electric field energy density

u_B: $(1/\mu_0) B^2$ magnetic field energy density

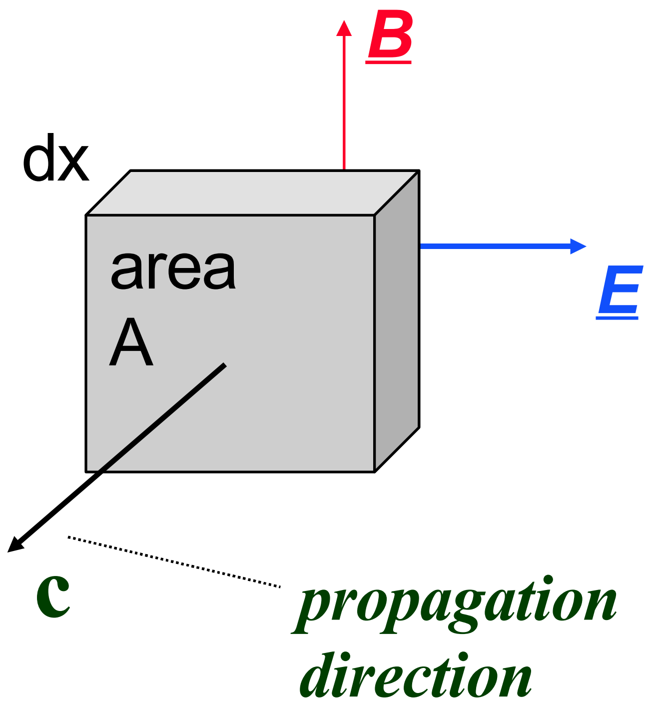
- The energy is put into the oscillating fields by the sources that generate them.

- This energy can then propagate to locations far away, at the velocity of light.

Energy in Electromagnetic Waves

Energy per unit volume is

$$u = u_E + u_B = \frac{1}{2}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2\right)$$



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Thus the energy, dU, in a box of area A and length dx is

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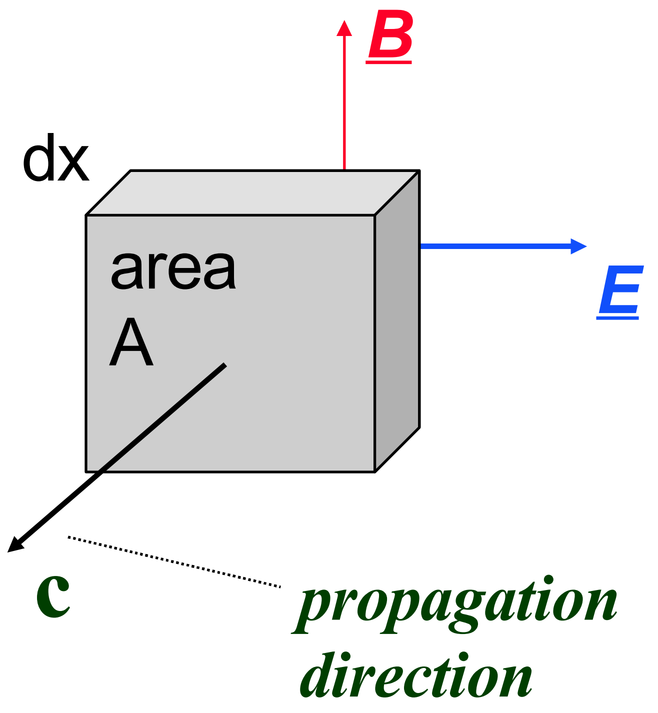

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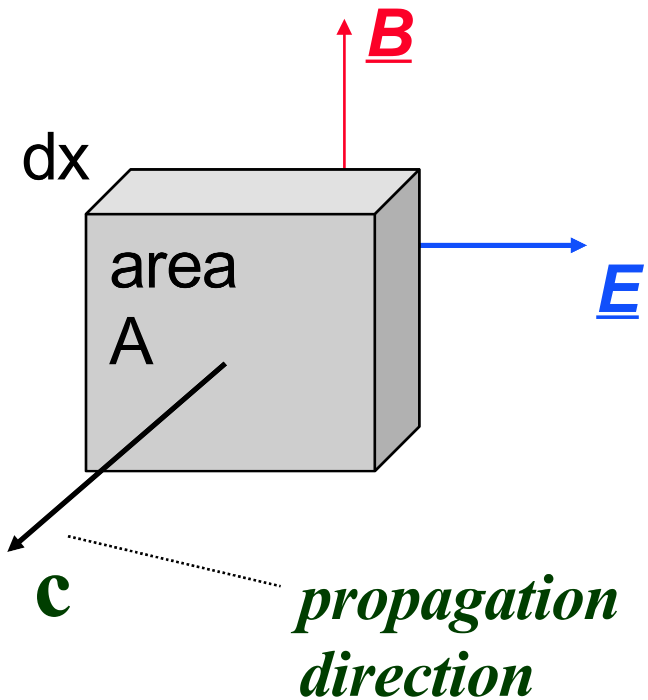

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$$dU = \frac{1}{2}(\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2) A dx$$

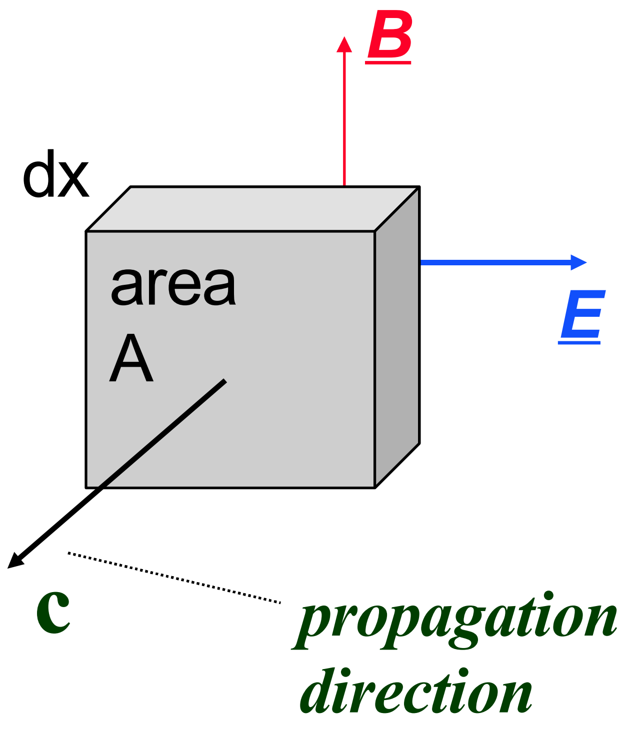
Let the length dx equal cdt. Then all of this energy leaves the box in time dt. Thus energy flows at the rate

$$\frac{dU}{dt} = \frac{1}{2}(\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2) Ac$$

Energy Flow in Electromagnetic Waves

Rate of energy flow:

$$\frac{dU}{dt} = \frac{c}{2}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2\right) A$$



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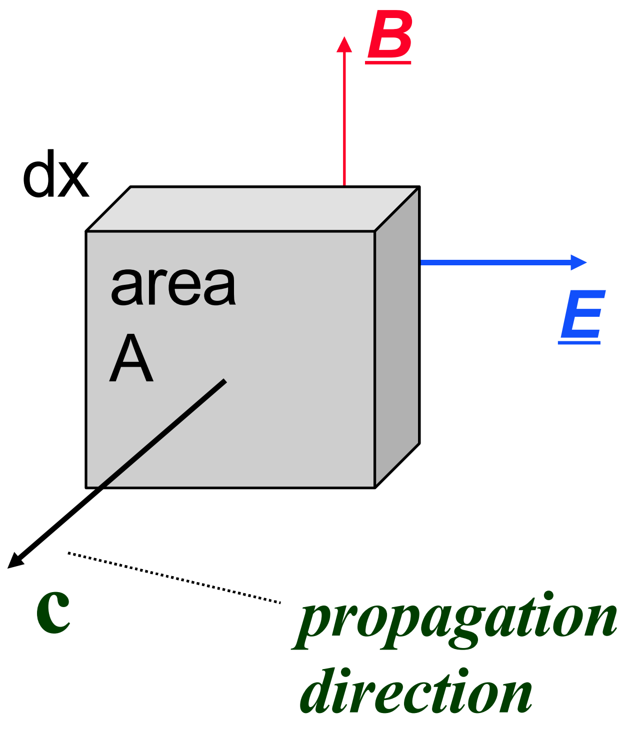

We define the intensity S, as the rate of energy flow per unit area:

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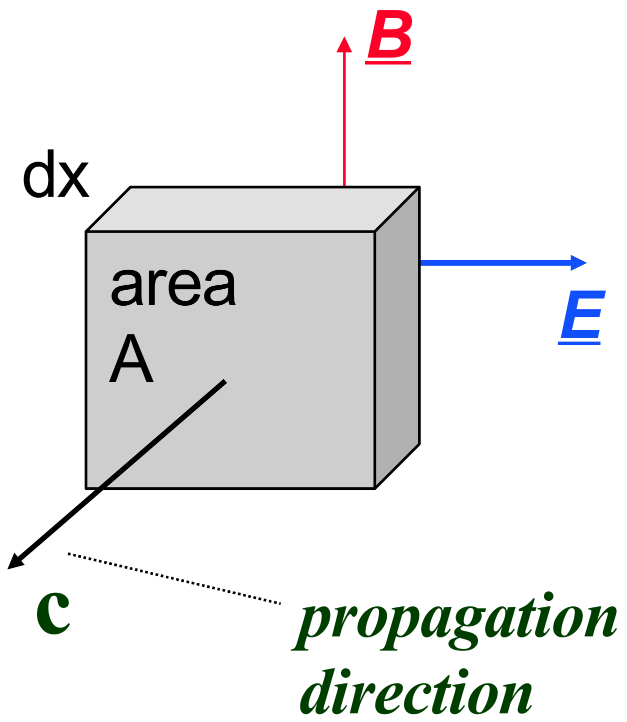

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$$S = \frac{c}{2}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2\right)$$

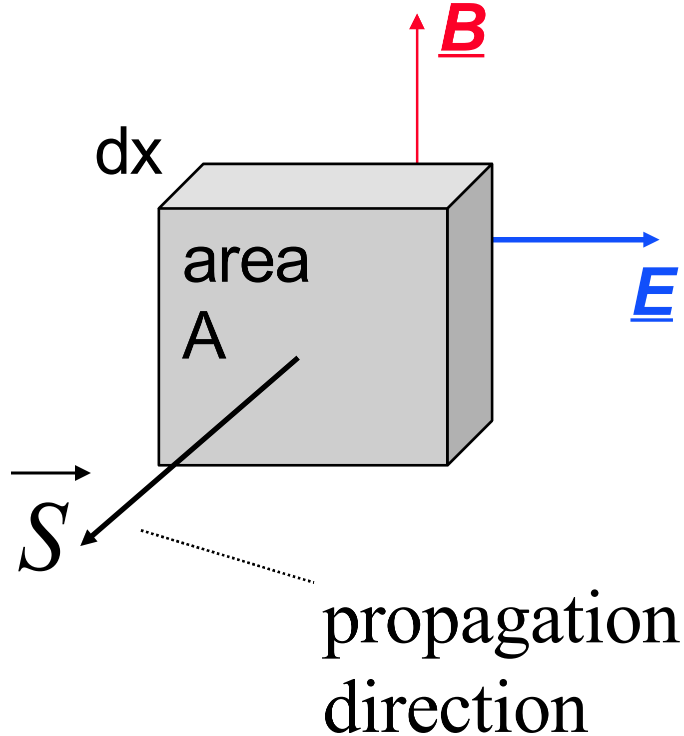
Rearranging by substituting E=cB and B=E/c, we get,

$$S = \frac{c}{2}\left(\varepsilon_0 cEB + \frac{1}{\mu_0 c} EB\right) = \frac{1}{2\mu_0}\left(\varepsilon_0 \mu_0 c^2 + 1\right)EB = \frac{EB}{\mu_0}$$

The Poynting Vector

In general, we find:

$$\underline{\mathbf{S}} = (1/\mu_0)\ \underline{\mathbf{E}} \times \underline{\mathbf{B}}$$

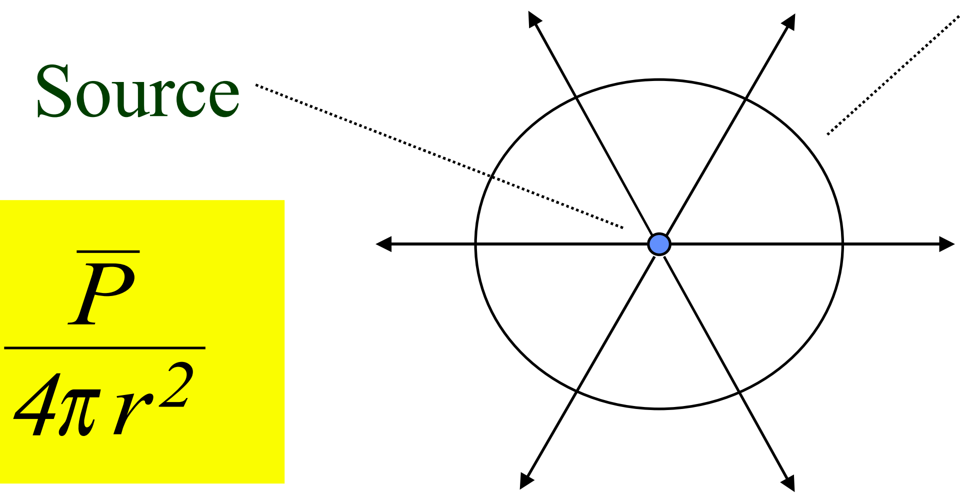


S is a vector that points in the direction of propagation of the wave and represents the rate of energy flow per unit area.
We call this the "Poynting vector".

Units of S are $Jm^{-2}\,s^{-1}$, or Watts/m^2.

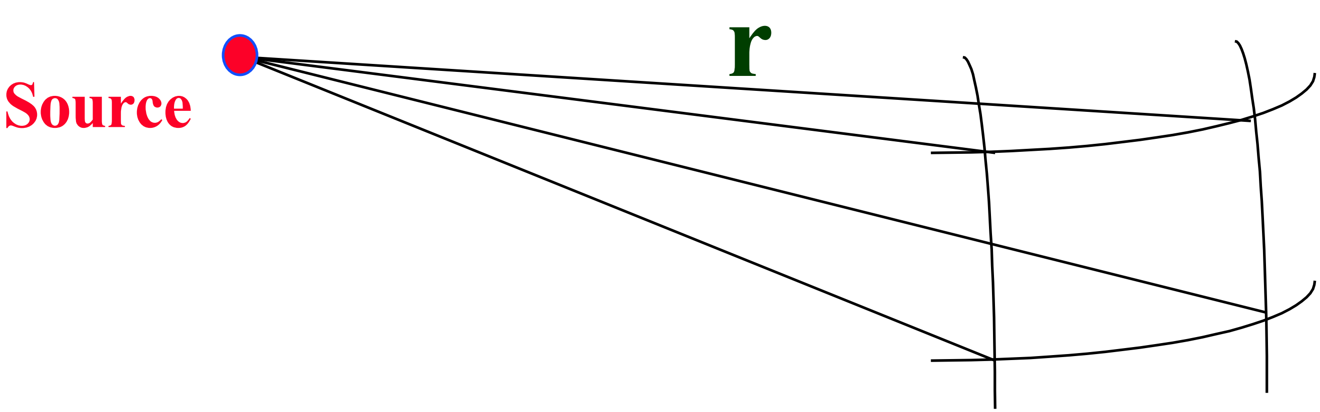
The Inverse-Square Dependence of <u>S</u>

A point source of light, or any radiation, spreads out in all directions:



Power, P, flowing through sphere is same for any radius.

$$\overline{S} = \frac{\overline{P}}{4\pi r^2}$$



$$Area \propto r^2$$

$$\overline{S} \propto \frac{1}{r^2}$$

Example:

An observer is 1.8 m from a point light source whose average power P= 250 W. Calculate the rms fields in the position of the observer.

$$\textit{Intensity of light at a distance } r \textit{ is} \quad S = P / 4\pi r^2$$

$$I = \frac{P}{4\pi r^2} = \frac{1}{\mu_0 c} E_{rms}^2$$

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$$\therefore\ E_{rms} = \sqrt{\frac{P\mu_0 c}{4\pi r^2}} = \sqrt{\frac{(250W)(4\pi\, 10^{-7}\, H/m)(3.10^8\, m/s)}{4\pi\,(1.8m)^2}}$$

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$$\therefore \; E_{rms} = 48V/m$$

$$\therefore \; B = \frac{E_{rms}}{c} = \frac{48V/m}{3.10^8 m/s} = 0.16\mu T$$

Wave Momentum and Radiation Pressure

Momentum and energy of a wave are related by, $p = U / c$.

Now, Force $= dp/dt = (dU/dt)/c$

pressure (radiation) = Force / unit area

$P = (dU/dt) / (A\,c) = S / c$

$\therefore$ Radiation Pressure $\Rightarrow P_{rad} = \dfrac{\bar{S}}{c}$

Example: **Serious proposals have been made to "sail" spacecraft to the outer solar system using the pressure of sunlight. How much sail area must a 1000 kg spacecraft have if its acceleration is to be 1 m/s^2 at the Earth's orbit? Make the sail reflective.**

Can ignore gravity.

Need $F=ma=(1000kg)(1\ m/s^2)=1000\ N$

This comes from pressure: $F=PA$, so $A=F/P$.

Here P is the radiation pressure of sunlight:

Sun's power = 4×10^{26} W, so $S=\text{power}/(4\pi r^2)$ gives

$$S = (4 \times 10^{26}\ W) / (4\pi(1.5x10^{11}m)^2) = 1.4kW/m^2.$$

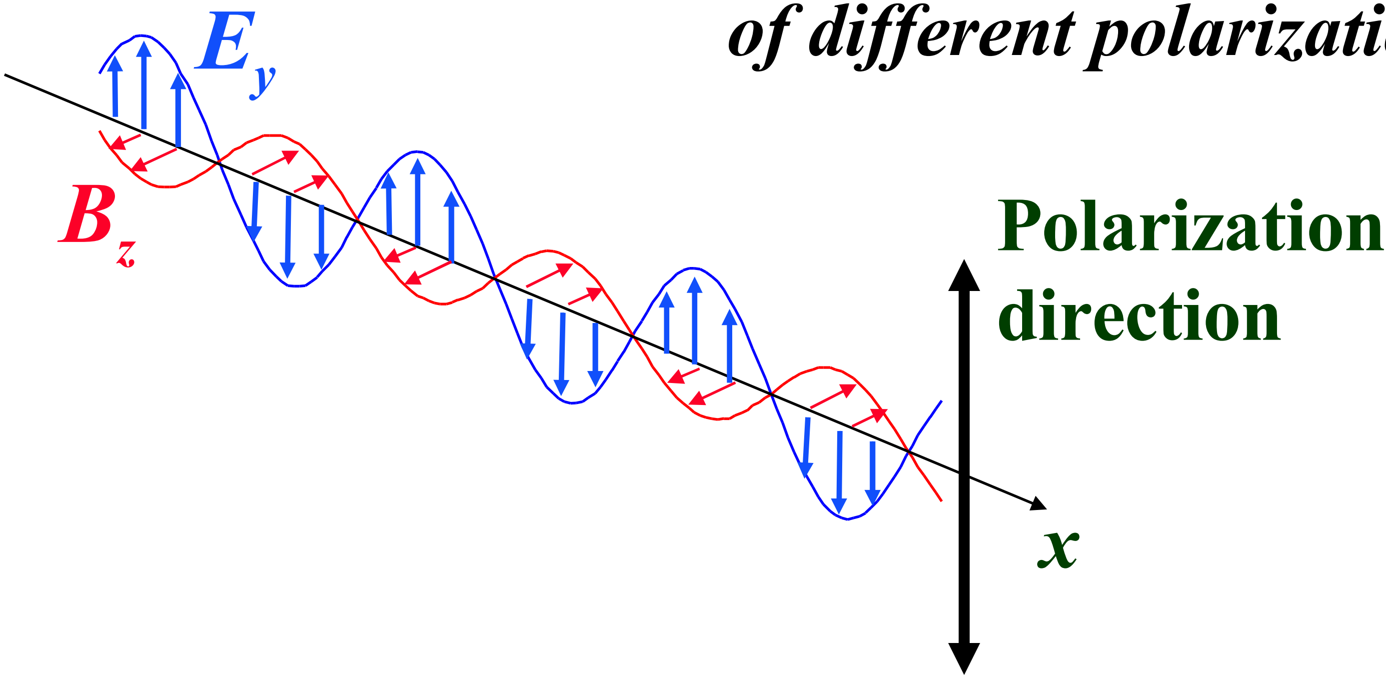
Thus the pressure due to this light, reflected, is:

$$P = 2S/c = 2(1400W/m^2) / 3x10^8 m/s = 9.4x10^{-6} N/m^2$$

Hence $A=1000N / 9.4x10^{-6}N/m^2 =1.0x10^8\ m^2 = 100\ km^2$

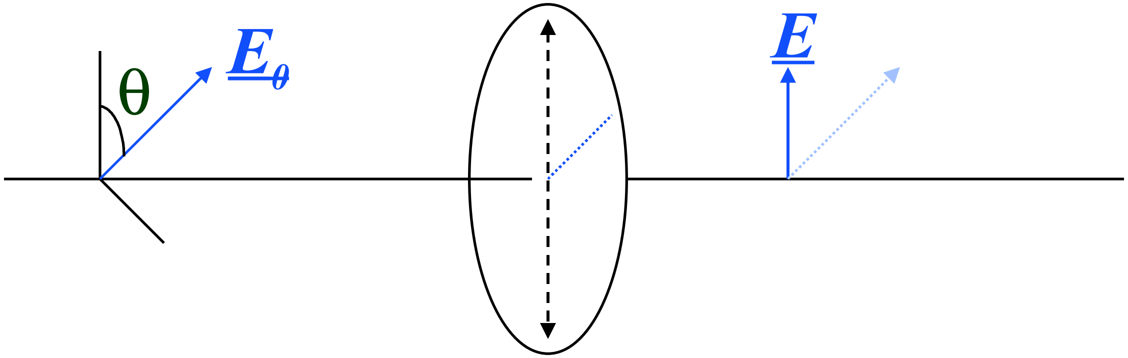
Polarization

The direction of polarization of a wave is the direction of the electric field. Most light is randomly polarized, which means it contains a mixture of waves of different polarizations.



Polarization

A **polarizer** ***lets through light of only one polarization:***



Transmitted light has its E in the direction of the polarizer's transmission axis.

$$E = E_0 \cos\theta$$

hence, $$S = S_0 \cos^2\theta$$

- Malus's Law