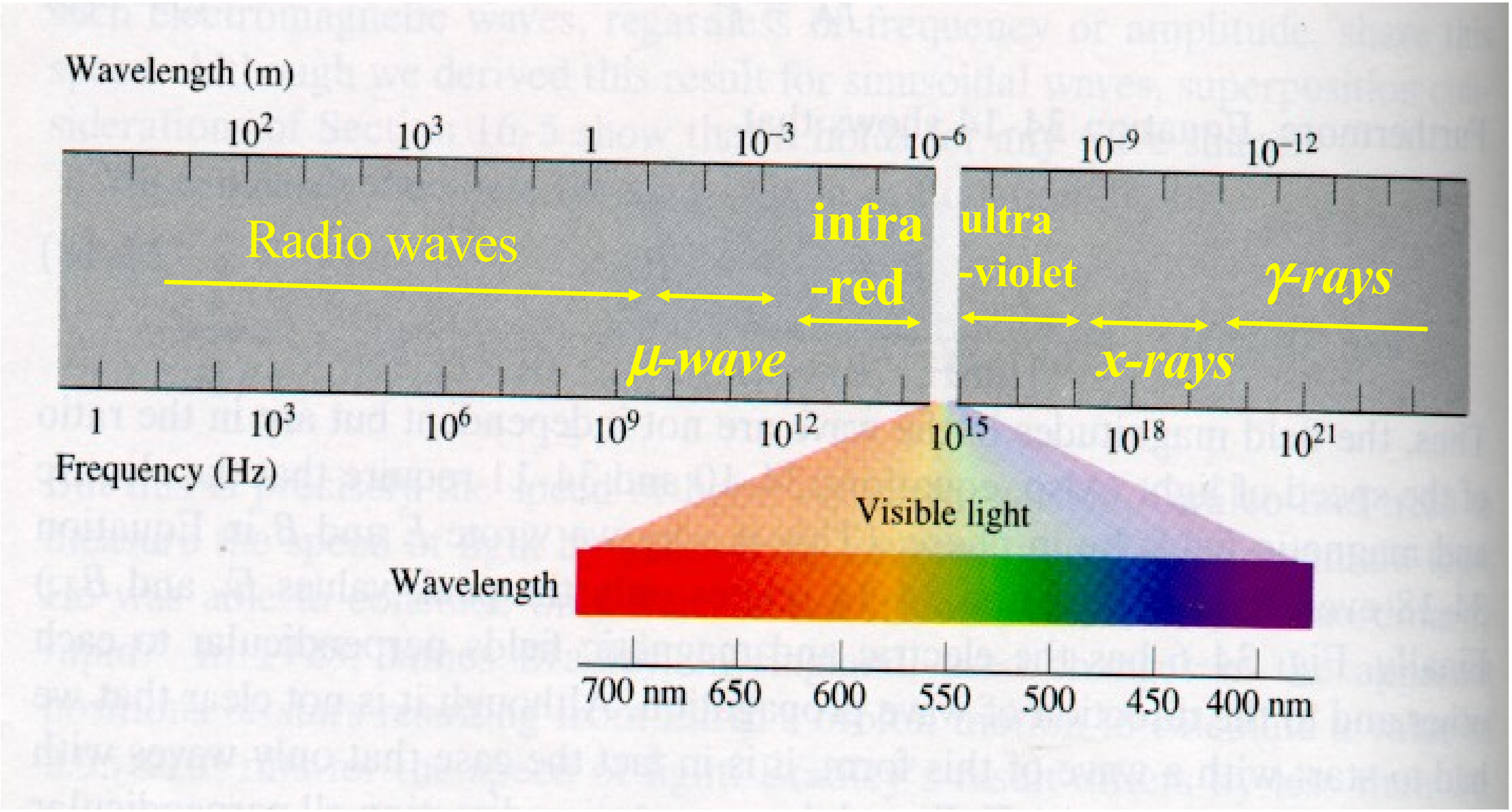
Chapter 32

The Laws of Electromagnetism
Maxwell's Equations
Displacement Current
Electromagnetic Radiation

The Electromagnetic Spectrum



The Equations of Electromagnetism (at this point …)

Gauss' Law for Electrostatics

$$\oint \underline{E} \bullet \underline{dA} = \frac{q}{\varepsilon_0}$$

Gauss' Law for Magnetism

$$\oint \underline{B} \bullet \underline{dA} = 0$$

Faraday's Law of Induction

$$\oint \underline{E} \bullet \underline{dl} = -\frac{d\Phi_B}{dt}$$

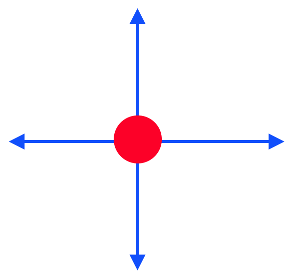
Ampere's Law

$$\oint \underline{B} \bullet \underline{dl} = \mu_0 I$$

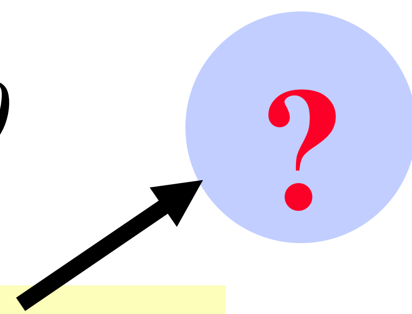
The Equations of Electromagnetism

Gauss's Laws ***..monopole..***

1 $$\oint \underline{E} \bullet \underline{dA} = \frac{q}{\varepsilon_0}$$



2 $$\oint \underline{B} \bullet \underline{dA} = 0$$



...there's no magnetic monopole....!!

The Equations of Electromagnetism

Faraday's Law

3 $$\oint \underline{E} \bullet \underline{dl} = -\frac{d\Phi_B}{dt}$$

.. if you change a magnetic field you induce an electric field.........

Ampere's Law

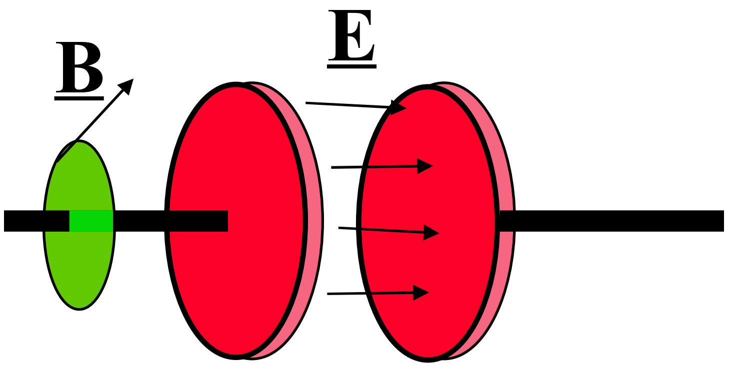
4 $$\oint \underline{B} \bullet \underline{dl} = \mu_0 I$$

.......is the reverse true..?

...lets take a look at charge flowing into a capacitor...

...when we derived Ampere's Law we assumed constant current...

$$\oint \underline{B} \bullet \underline{dl} = \mu_0 I$$



...lets take a look at charge flowing into a capacitor...

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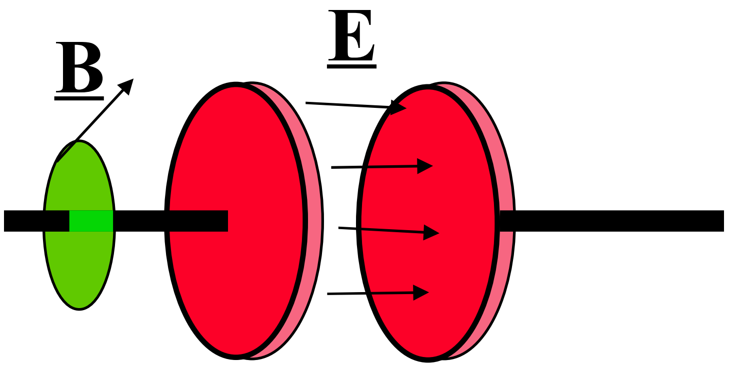
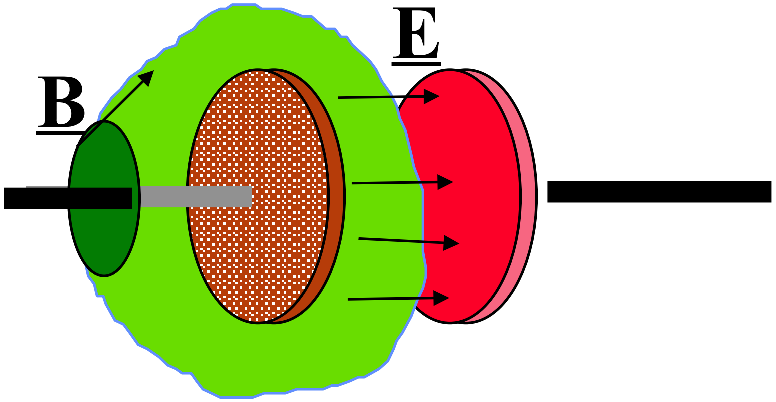
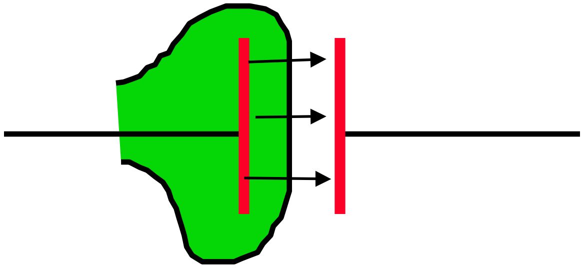
$$\oint \underline{B} \bullet \underline{dl} = \mu_0 I$$



.. if the loop encloses one plate of the capacitor..there is a problem … I = 0

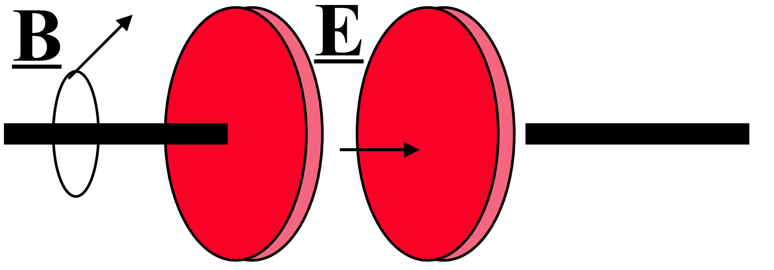


Side view: (Surface is now like a bag:)



Maxwell solved this problem by realizing that....

Inside the capacitor there must be an induced magnetic field...



How?.

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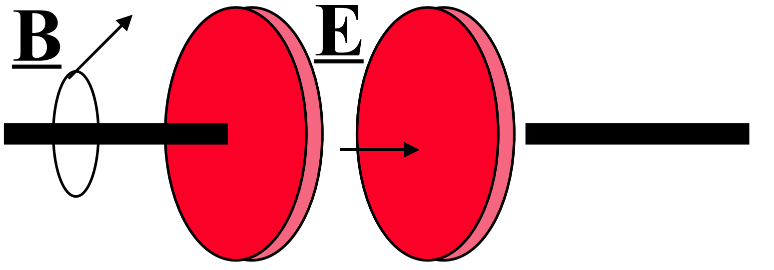
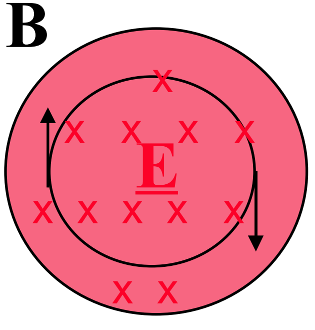


How?. Inside the capacitor there is a changing E ⇒



A changing electric field induces a magnetic field

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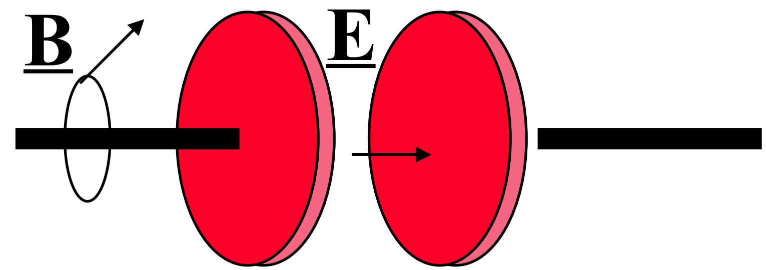

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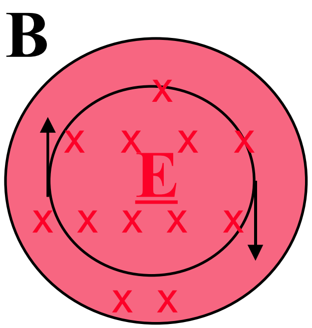

A changing electric field induces a magnetic field

$$\oint \underline{B} \bullet \underline{dl} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} = \mu_0 I_d$$

where I_d is called the ***displacement current***

Maxwell solved this problem by realizing that....

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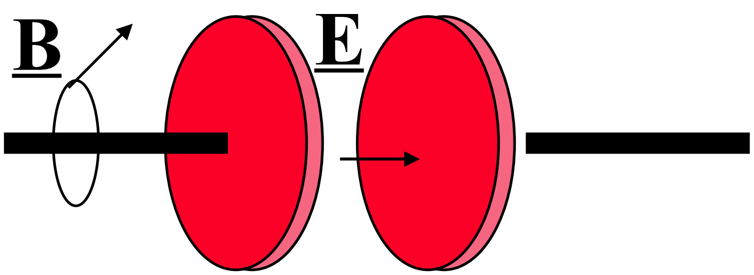

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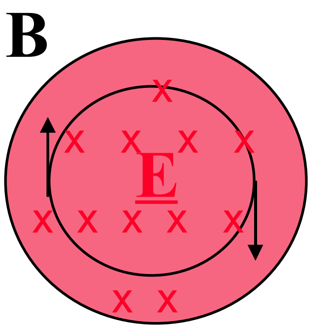

A changing electric field induces a magnetic field

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where I_d is called the <u>displacement current</u>

Therefore, Maxwell's revision of Ampere's Law becomes....

$$\oint \underline{B} \bullet \underline{dl} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Derivation of Displacement Current

For a capacitor, $q = \varepsilon_0 EA$ and $I = \dfrac{dq}{dt} = \varepsilon_0 \dfrac{d(EA)}{dt}$.

Now, the electric flux is given by EA, so: $I = \varepsilon_0 \dfrac{d(\Phi_E)}{dt}$,
where this current , not being associated with charges, is called the “Displacement current”, I_d.

Hence: $$I_d = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

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Hence:

$$I_d = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

and:

$$\oint \underline{B} \bullet \underline{dl} = \mu_0 (I + I_d)$$

$$\Rightarrow \oint \underline{B} \bullet \underline{dl} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Maxwell's Equations of Electromagnetism

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Ampere's Law $$\oint \underline{B} \bullet \underline{dl} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Maxwell's Equations of Electromagnetism in Vacuum (no charges, no masses)

Consider these equations in a vacuum.....

......no mass, no charges. no currents.....

$$\oint \underline{E} \bullet \underline{dA} = \frac{q}{\varepsilon_0} \longrightarrow \oint \underline{E} \bullet \underline{dA} = 0$$

$$\oint \underline{B} \bullet \underline{dA} = 0 \longrightarrow \oint \underline{B} \bullet \underline{dA} = 0$$

$$\oint \underline{E} \bullet \underline{dl} = -\frac{d\Phi_B}{dt} \longrightarrow \oint \underline{E} \bullet \underline{dl} = -\frac{d\Phi_B}{dt}$$

$$\oint \underline{B} \bullet \underline{dl} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \longrightarrow \oint \underline{B} \bullet \underline{dl} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Maxwell's Equations of Electromagnetism in Vacuum (no charges, no masses)

$$\oint \underline{E} \bullet \underline{dA} = 0$$

$$\oint \underline{B} \bullet \underline{dA} = 0$$

$$\oint \underline{E} \bullet \underline{dl} = -\frac{d\Phi_B}{dt}$$

$$\oint \underline{B} \bullet \underline{dl} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Electromagnetic Waves

Faraday's law: **dB/dt → electric field**
Maxwell's modification of Ampere's law
dE/dt → magnetic field

$$\oint \underline{B} \bullet \underline{dl} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \qquad \oint \underline{E} \bullet \underline{dl} = -\frac{d\Phi_B}{dt}$$

These two equations can be solved simultaneously.

The result is:

$$\underline{\mathbf{E}}(x, t) = E_P \sin(kx - \omega t)\ \hat{j}$$

$$\underline{\mathbf{B}}(x, t) = B_P \sin(kx - \omega t)\ \hat{z}$$

Electromagnetic Waves

$$\oint \underline{B} \bullet \underline{dl} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

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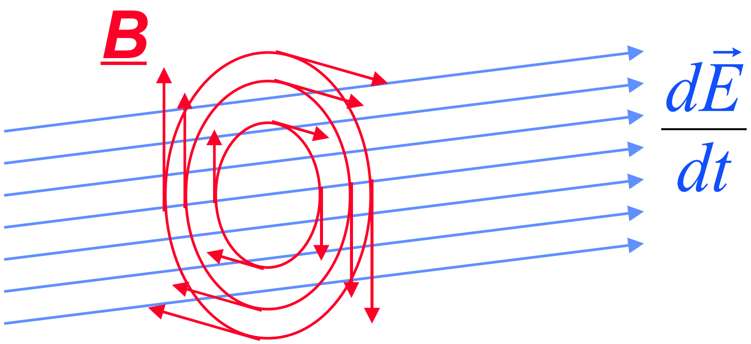

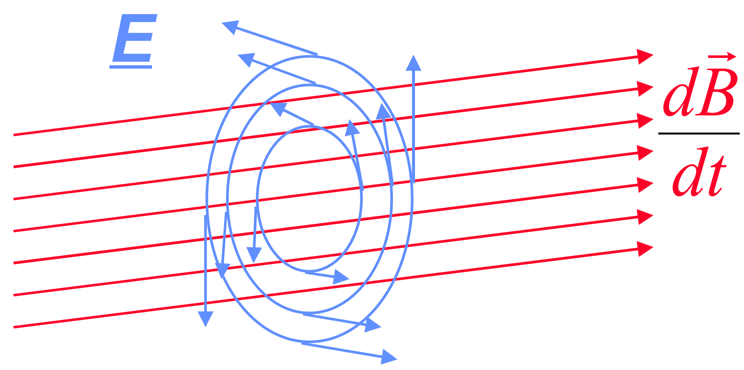

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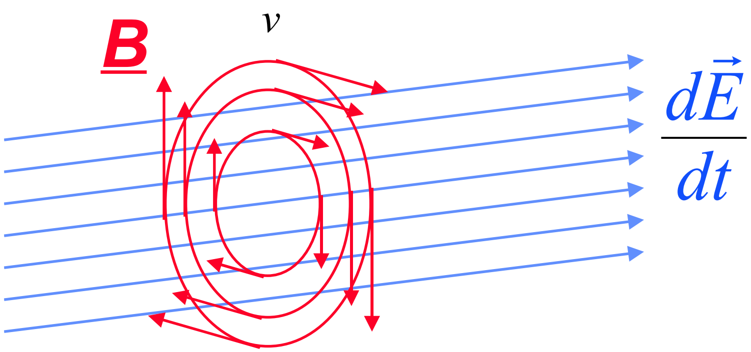
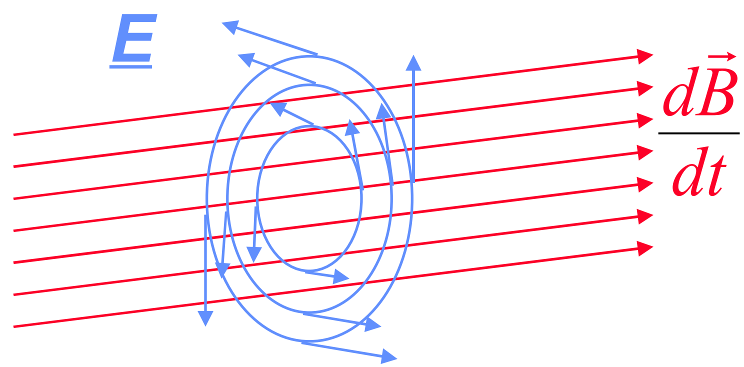


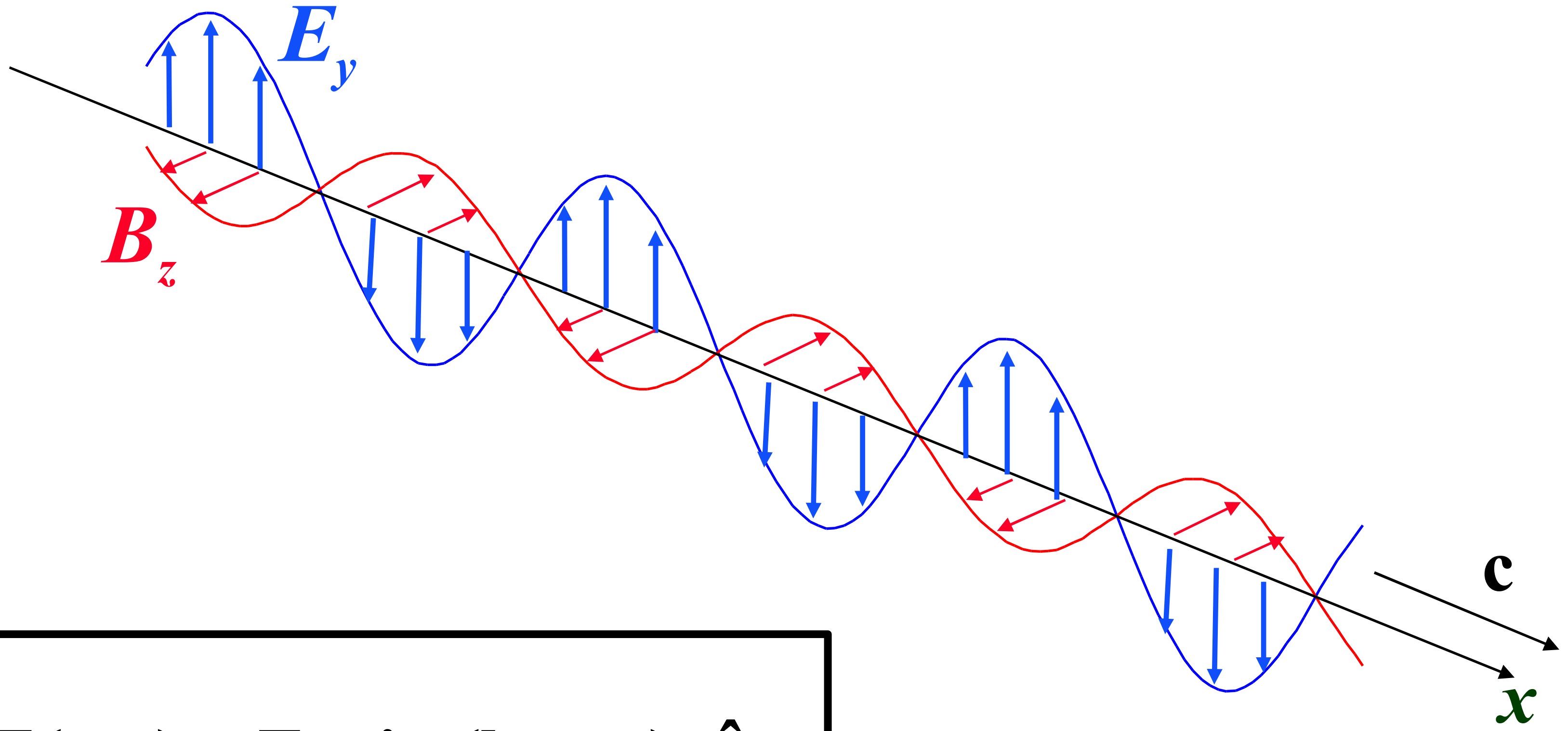


Special case..PLANE WAVES... $\vec{E} = E_y(x,t)\hat{j}$ $\vec{B} = B_z(x,t)\hat{k}$

satisfy the wave equation $$\frac{\partial^2\psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2}$$

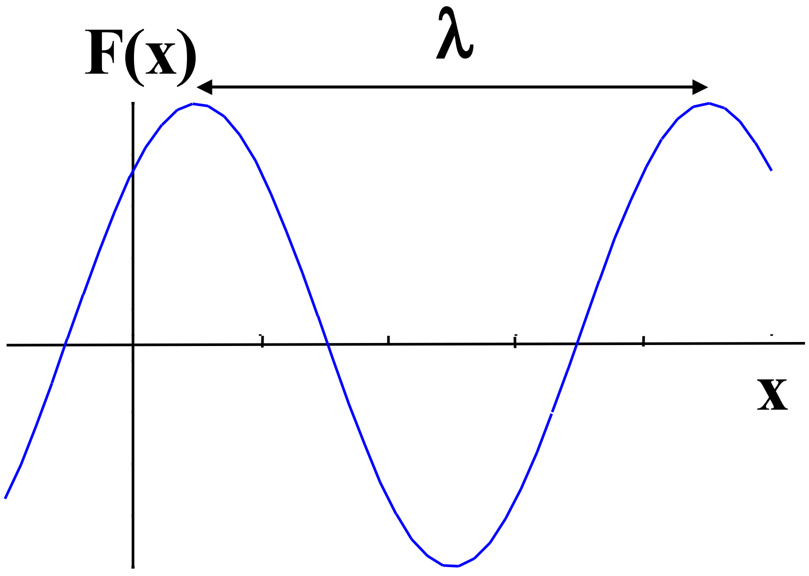
Maxwell's Solution → $\psi = A\sin(\omega t + \phi)$

Plane Electromagnetic Waves



$$\underline{E}(x, t) = E_P \sin(kx - \omega t)\ \hat{j}$$

$$\underline{B}(x, t) = B_P \sin(kx - \omega t)\ \hat{z}$$



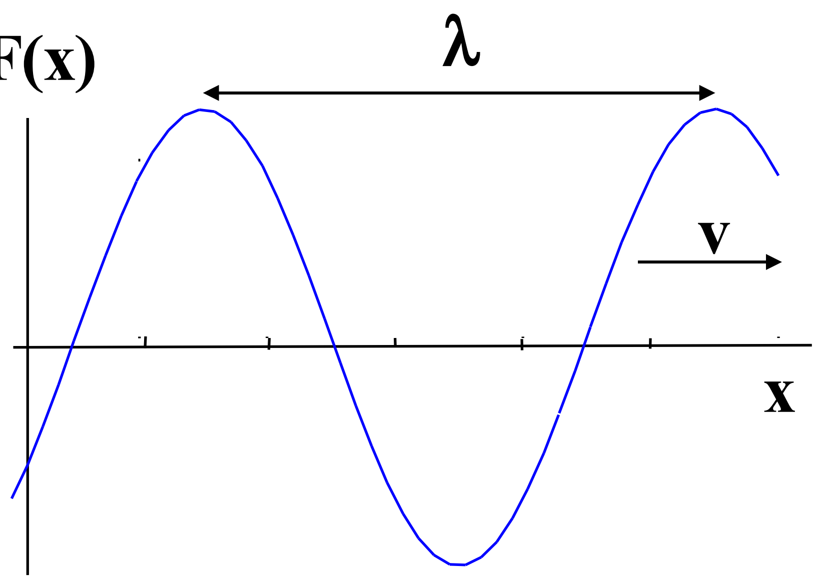
Static wave

$F(x) = F_P \sin(kx + \phi)$

$k = 2\pi / \lambda$

k = wavenumber

λ = wavelength



Moving wave

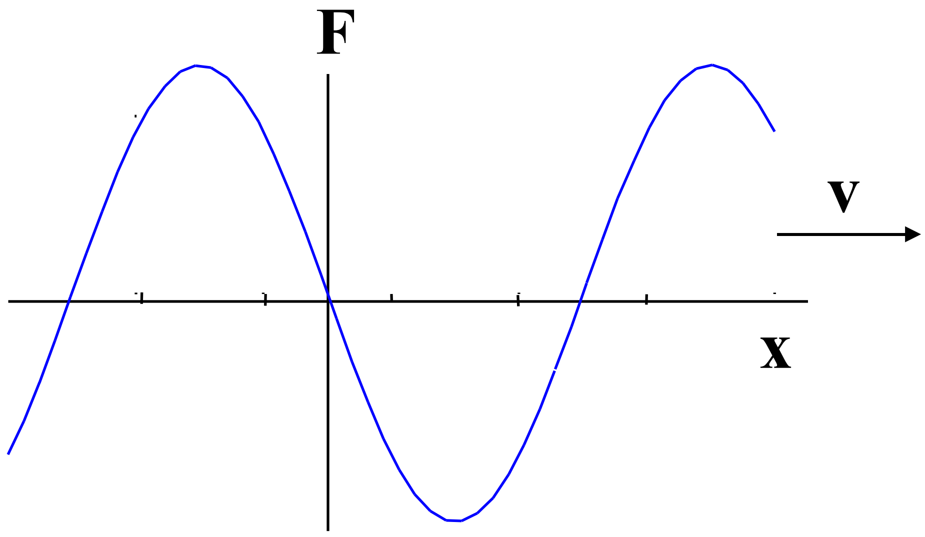
$F(x, t) = F_P \sin(kx - \omega t)$

$\omega = 2\pi / f$

ω = angular frequency

f = frequency

$v = \omega / k$



Moving wave

$$F(x, t) = F_P \sin (kx - \omega t)$$

What happens at x = 0 as a function of time?

$F(0, t) = F_P \sin (-\omega t)$

For x = 0 and t = 0 $\Rightarrow F(0, 0) = F_P \sin (0)$

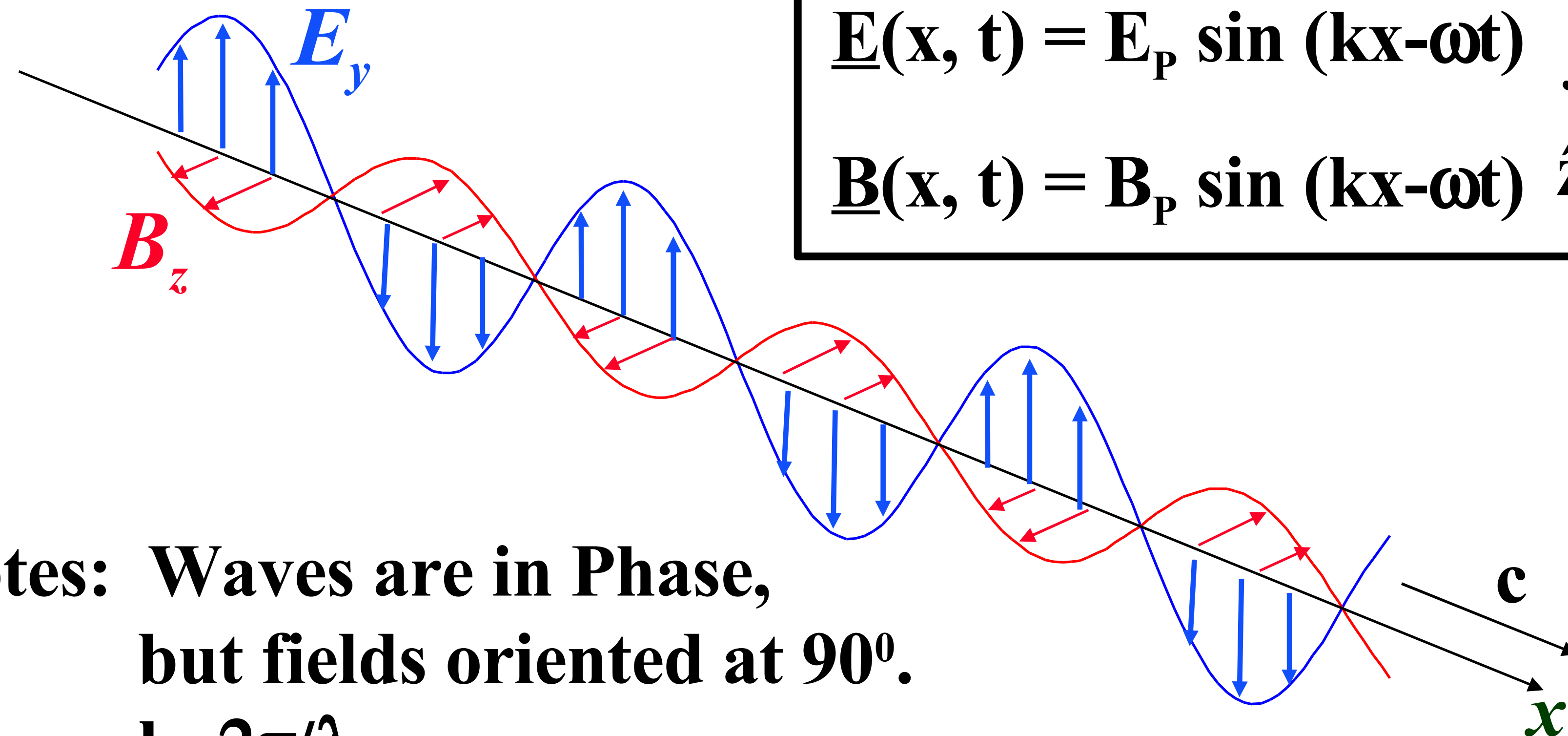
For x = 0 and t = t $\Rightarrow F(0, t) = F_P \sin (0 - \omega t) = F_P \sin (-\omega t)$

This is equivalent to: $kx = -\omega t \Rightarrow x = -(\omega/k)\, t$

F(x=0) at time t is the same as F[x=-(ω/k)t] at time 0

The wave moves to the right with speed ω/k

Plane Electromagnetic Waves



$\underline{E}(x, t) = E_P \sin(kx - \omega t)\ \hat{j}$

$\underline{B}(x, t) = B_P \sin(kx - \omega t)\ \hat{z}$

Notes: Waves are in Phase,
but fields oriented at 90°.
$k = 2\pi/\lambda$.
Speed of wave is $c = \omega/k$ ($= f\lambda$)

$c = 1/\sqrt{\varepsilon_0 \mu_0} = 3 \times 10^8 m/s$

At all times E=cB.