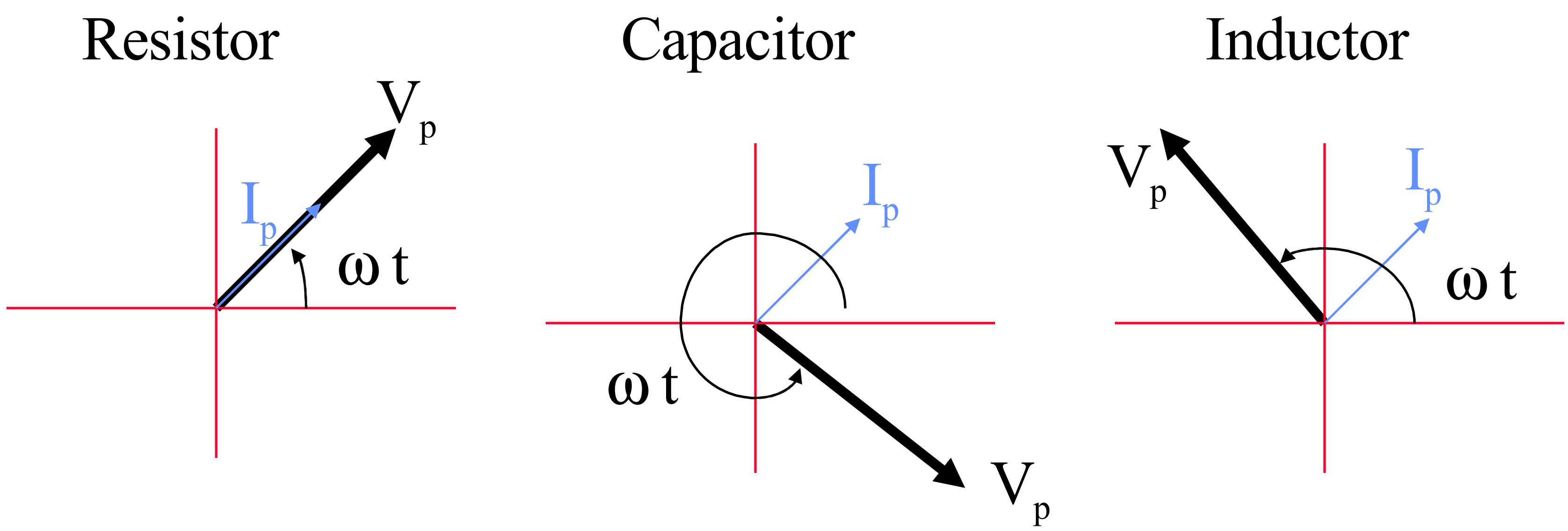
Alternating Current Circuits

Chapter 33

(continued)

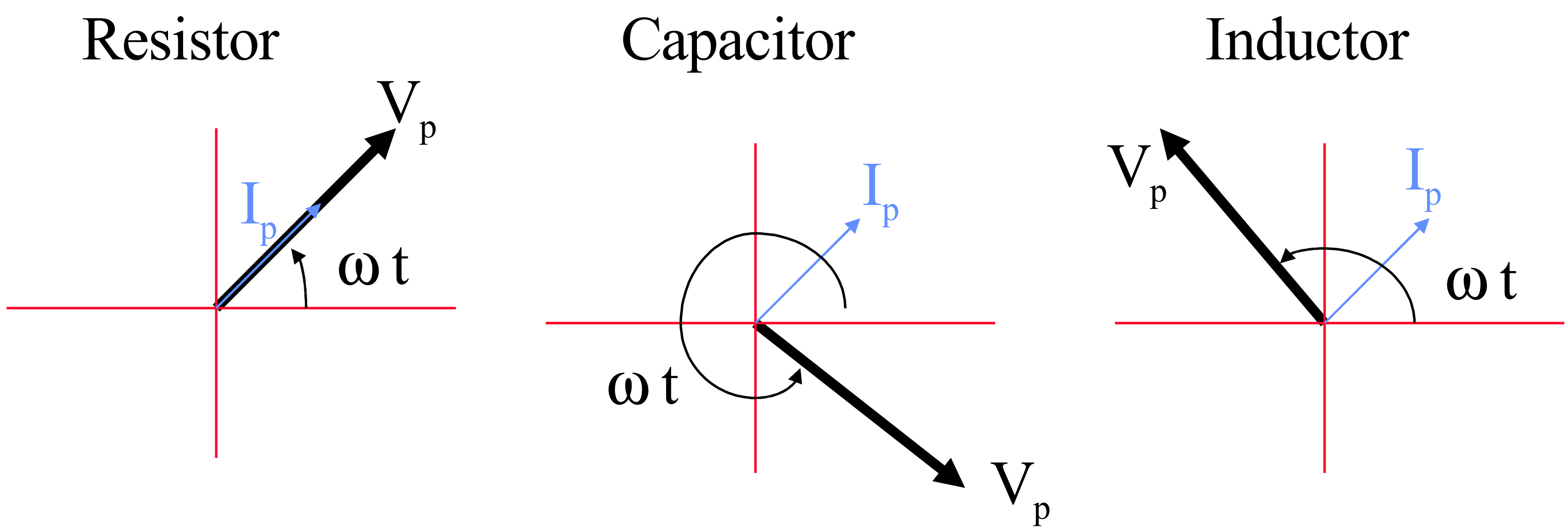
Phasor Diagrams

A phasor is an arrow whose length represents the amplitude of an AC voltage or current.
The phasor rotates counterclockwise about the origin with the angular frequency of the AC quantity.
Phasor diagrams are useful in solving complex AC circuits.

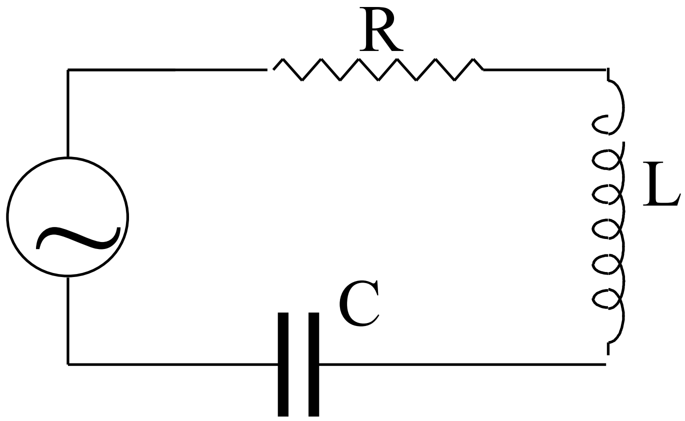


Reactance - Phasor Diagrams

CIRCUIT ELEMENT	REACTANCE	AMPLITUDE RELATION	PHASE RELATION
resistor	R	$I_o = V_o/R$	I,V in phase
capacitor	$X_c = 1/\omega C$	$I_o = V_o/X_c$	I leads V by 90°
inductor	$X_L = \omega L$	$I_o = V_o/X_L$	V leads I by 90°



“Impedance” of an AC Circuit

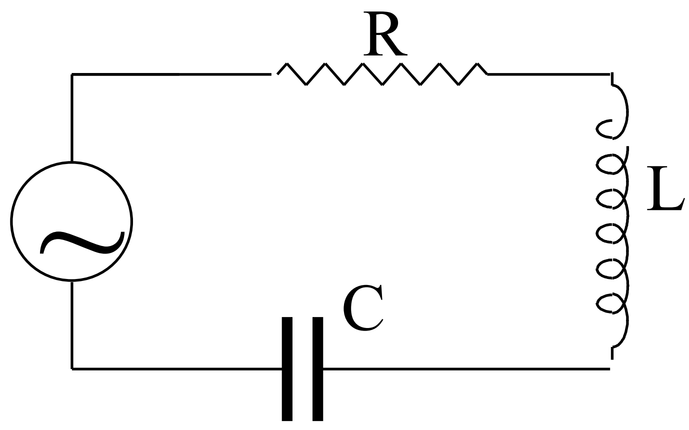


The impedance, Z, of a circuit relates peak current to peak voltage:

$$I_p = \frac{V_p}{Z}$$

(Units: OHMS)

"*Impedance*" of an AC Circuit

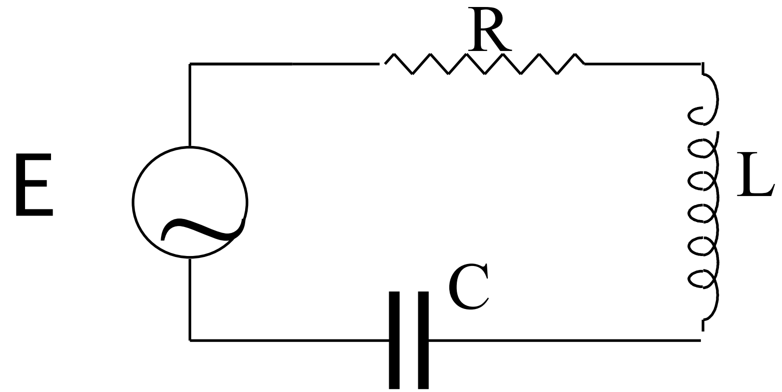


The impedance, Z, of a circuit relates peak current to peak voltage:

$$I_p = \frac{V_p}{Z}$$ (Units: OHMS)

(This is the AC equivalent of Ohm's law.)

Impedance of an RLC Circuit

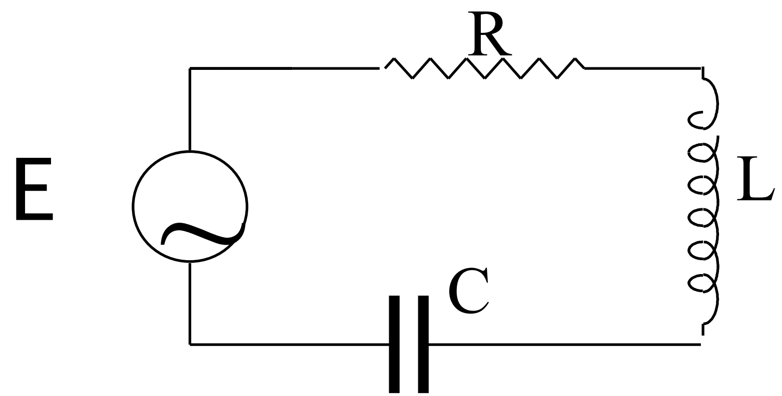


As in DC circuits, we can use the loop method:

$$E - V_R - V_C - V_L = 0$$

I is same through all components.

Impedance of an RLC Circuit



As in DC circuits, we can use the loop method:

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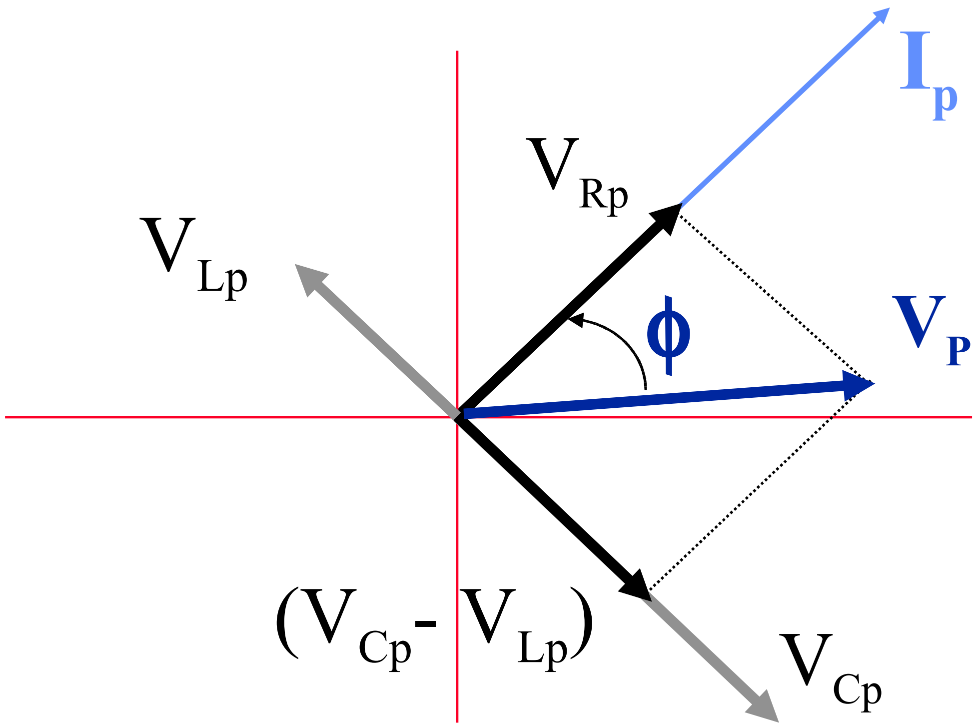
BUT: Voltages have different PHASES

$\Rightarrow$ they add as PHASORS.

Phasors for a Series RLC Circuit



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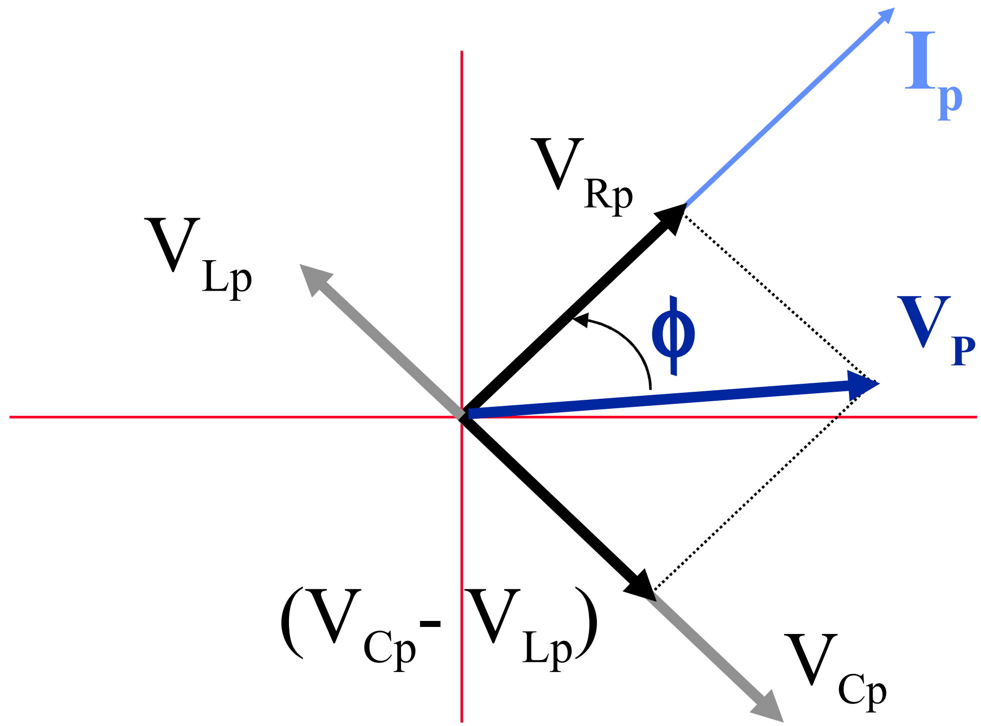

By Pythagoras' theorem:

$$(V_P)^2 = [\,(V_{Rp})^2 + (V_{Cp} - V_{Lp})^2\,]$$

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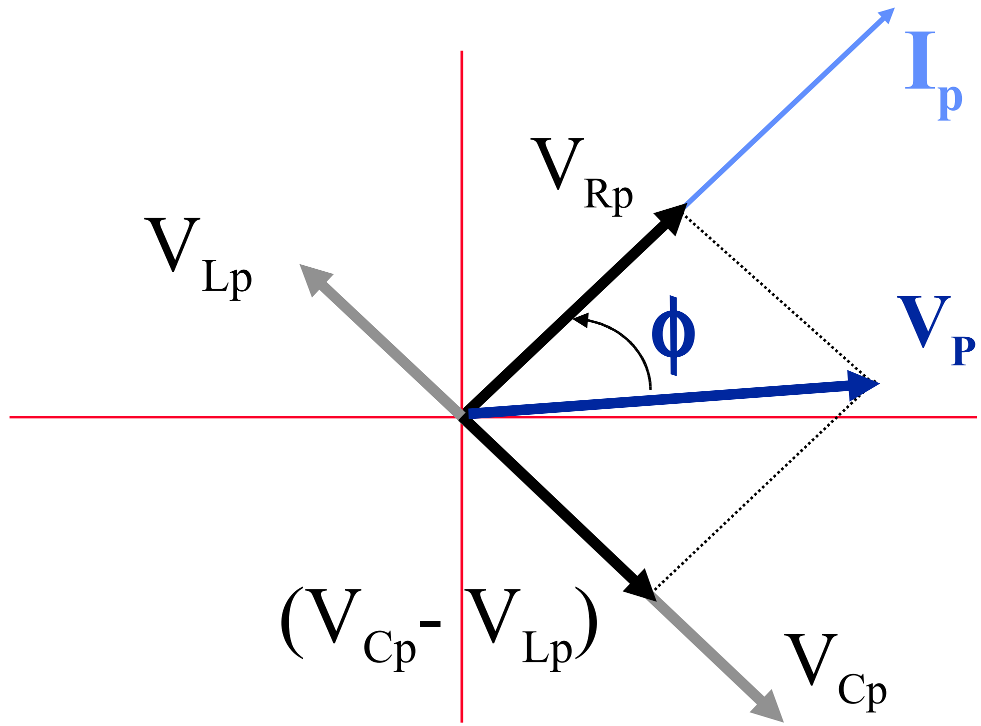

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$$= I_p^2 R^2 + (I_p X_C - I_p X_L)^2$$

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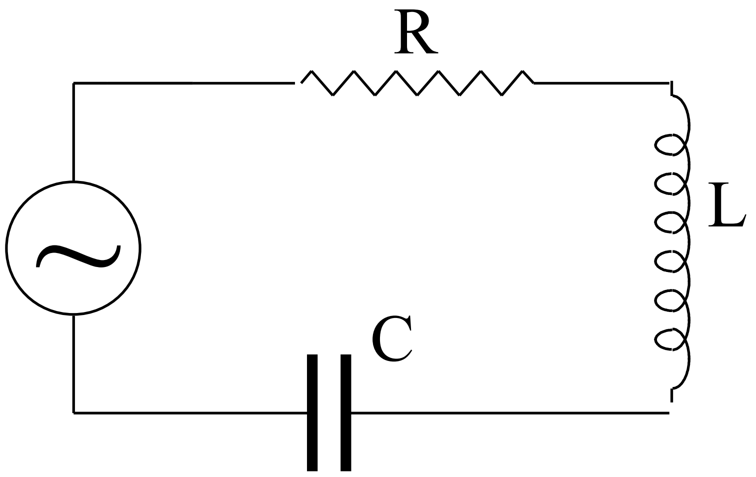

Solve for the current:

$$I_p = \frac{V_p}{\sqrt{R^2 + (X_c - X_L)^2}} = \frac{V_p}{Z}$$

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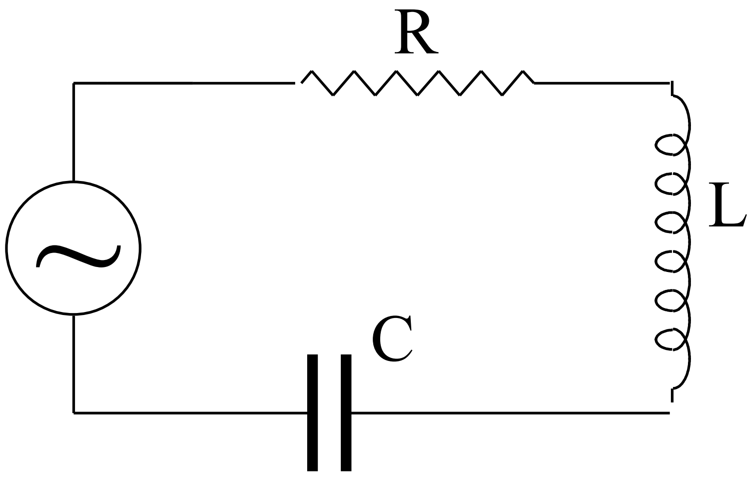

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Impedance:

$$Z = \sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2}$$

Impedance of an RLC Circuit

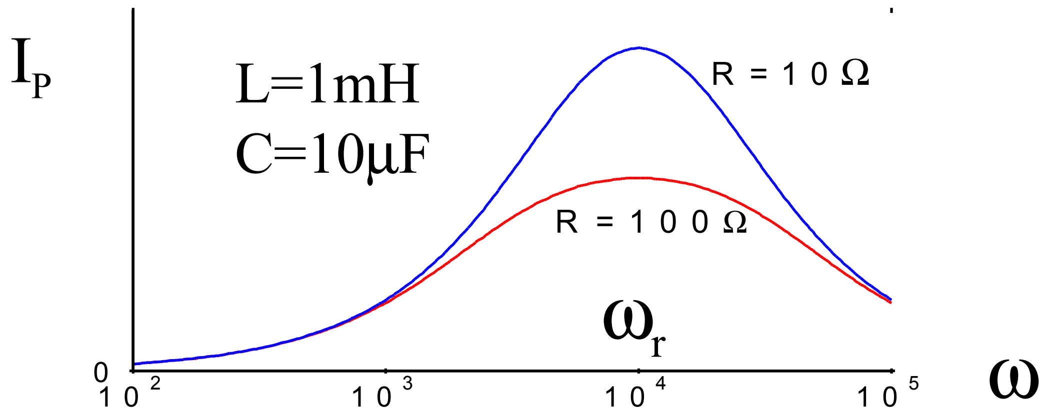
$$I_p = \frac{V_p}{Z}$$

$$Z = \sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2}$$

The current's magnitude depends on the driving frequency. When Z is a minimum, the current is a maximum. This happens at a resonance frequency:

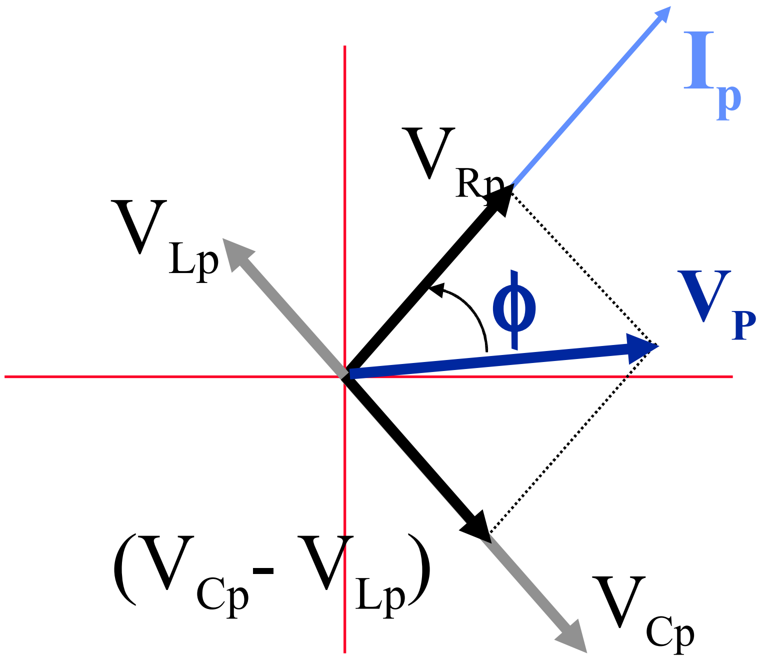
The circuit hits resonance when $1/\omega C - \omega L = 0$: $\omega_r = 1/\sqrt{LC}$

When this happens the capacitor and inductor cancel each other and the circuit behaves purely resistively: $I_P = V_P/R$.



The current dies away at both low and high frequencies.

Phase in an RLC Circuit



We can also find the phase:

$$\tan\phi = (V_{Cp} - V_{Lp}) / V_{Rp}$$

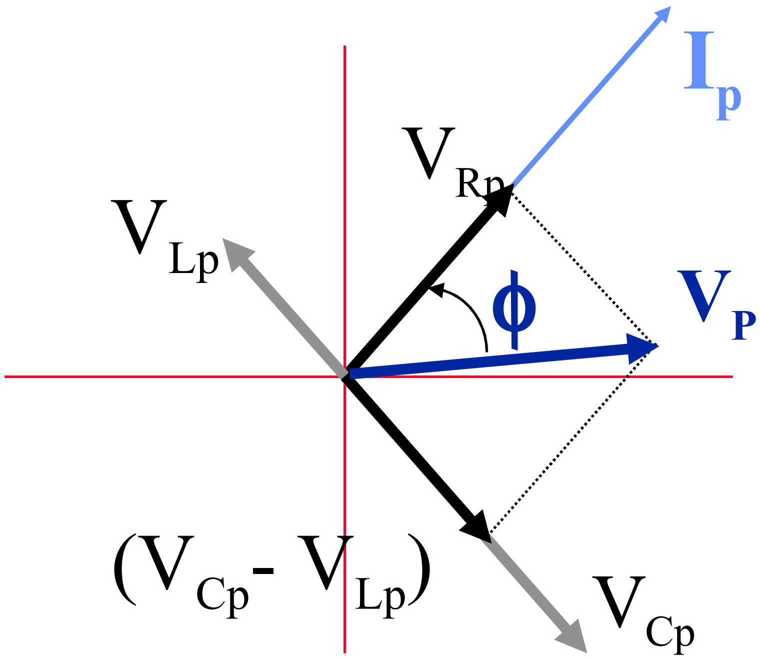
or;

$$\tan\phi = (X_C - X_L)/R.$$

or

$$\tan\phi = (1/\omega C - \omega L) / R$$

Phase in an RLC Circuit



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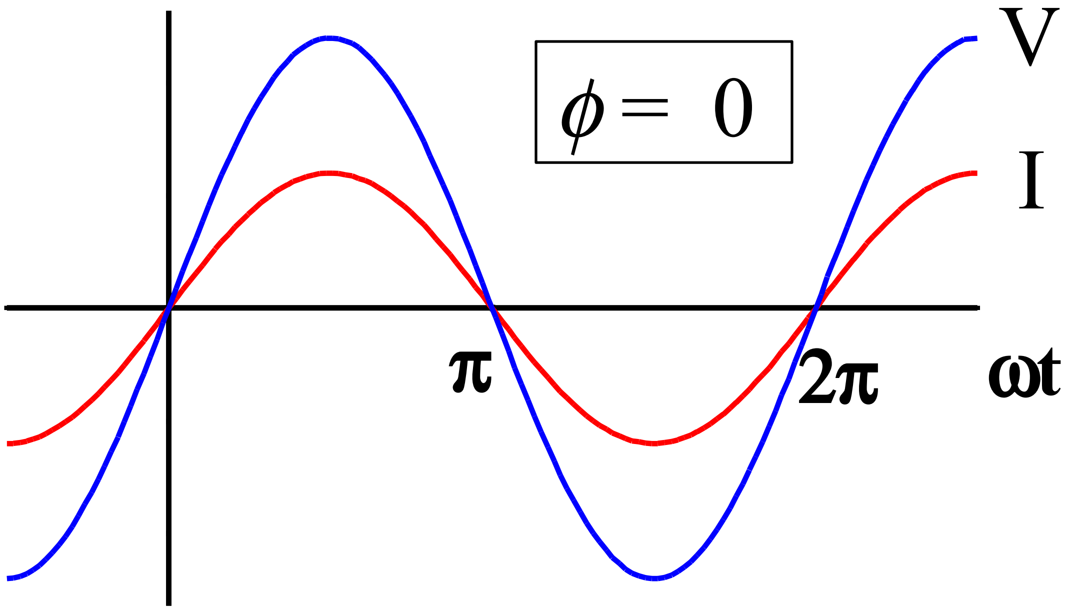
More generally, in terms of impedance:

$$\cos\phi = R/Z$$

At resonance the phase goes to zero (when the circuit becomes purely resistive, the current and voltage are in phase).

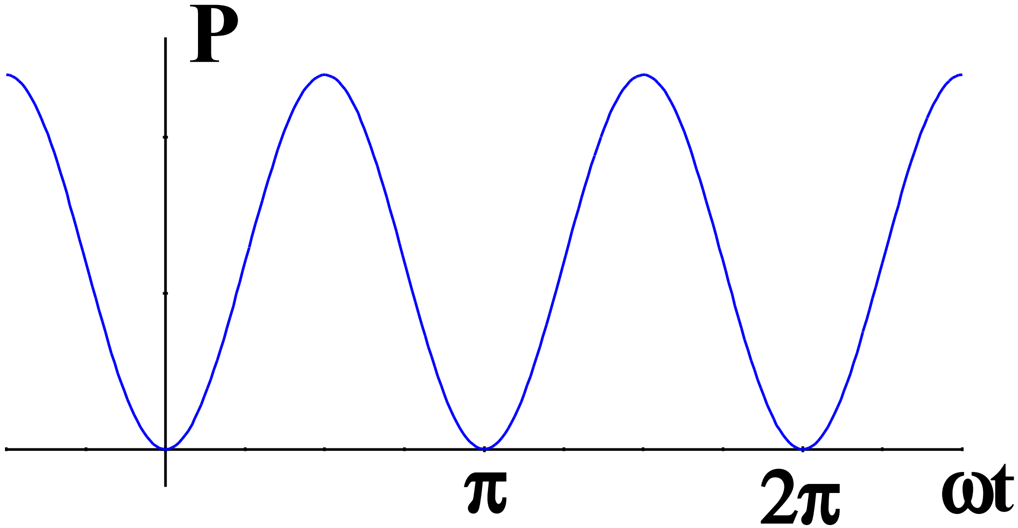
Power in an AC Circuit



V(t) = V_P sin (ωt)

I(t) = I_P sin (ωt)

(This is for a purely ***resistive*** circuit.)



P(t) = IV = $I_P V_P \sin^2(\omega t)$

Note this oscillates *twice as fast.*

Power in an AC Circuit

The power is P=IV. Since both I and V vary in time, so does the power: P is a function of time.

Use, $V = V_p \sin(\omega t)$ and $I = I_p \sin(\omega t+\phi)$:

$$P(t) = I_p V_p \sin(\omega t) \sin(\omega t+\phi)$$

This wiggles in time, usually very fast. What we usually care about is the time average of this:

$$\langle P \rangle = \frac{1}{T}\int_0^T P(t)\,dt \qquad (T=1/f)$$

Power in an AC Circuit

Now: $$\sin(\omega t + \phi) = \sin(\omega t)\cos\phi + \cos(\omega t)\sin\phi$$

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which we usually write as $\left\langle P \right\rangle = I_{rms} V_{rms} cos\phi$

Power in an AC Circuit

$$\langle P \rangle = I_{rms} V_{rms} \cos\phi$$

(ϕ goes from -90^0 to 90^0, so the average power is positive)

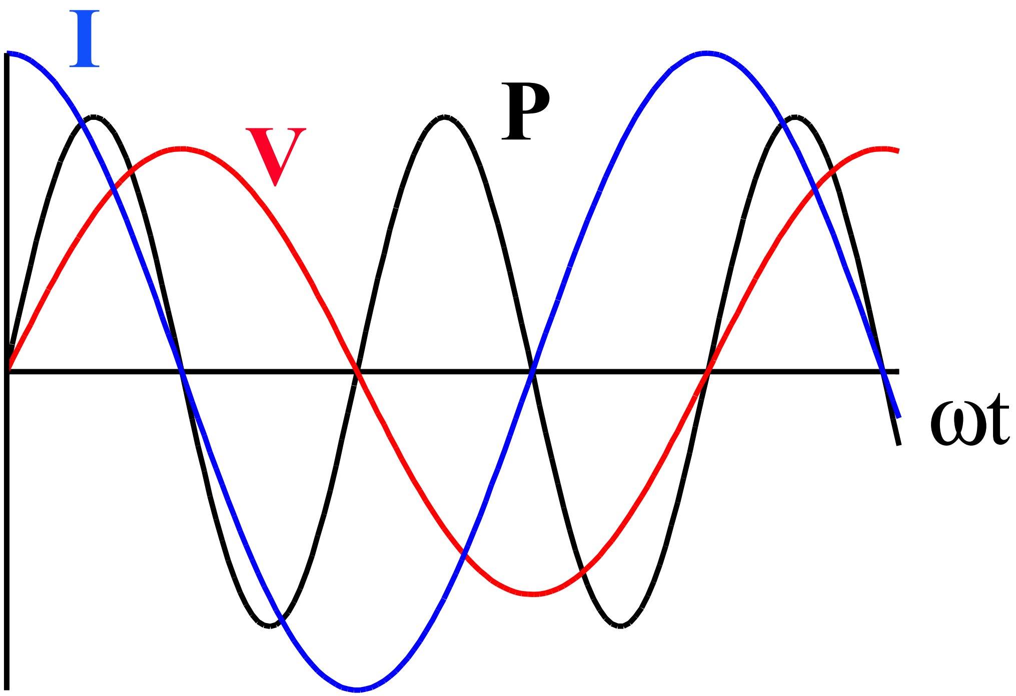
$\cos(\phi)$ is called the ***power factor***.

For a purely resistive circuit the power factor is 1.
When R=0, $\cos(\phi)=0$ (energy is traded but not dissipated).
Usually the power factor depends on frequency.

Power in an AC Circuit

$$\langle P \rangle = I_{rms} V_{rms} \cos\phi$$

What if ϕ is not zero?



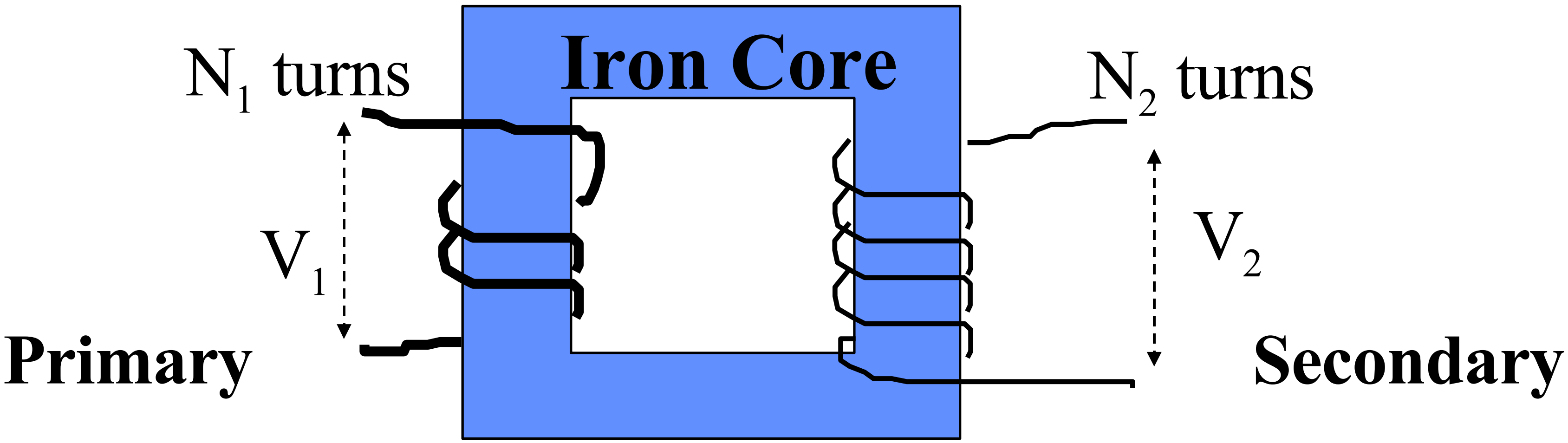
Here I and V are 90^0 out of phase. ($\phi = 90^0$) (It is purely *reactive*)

The *time average* of P is zero.

Transformers

Transformers use mutual inductance to change voltages:

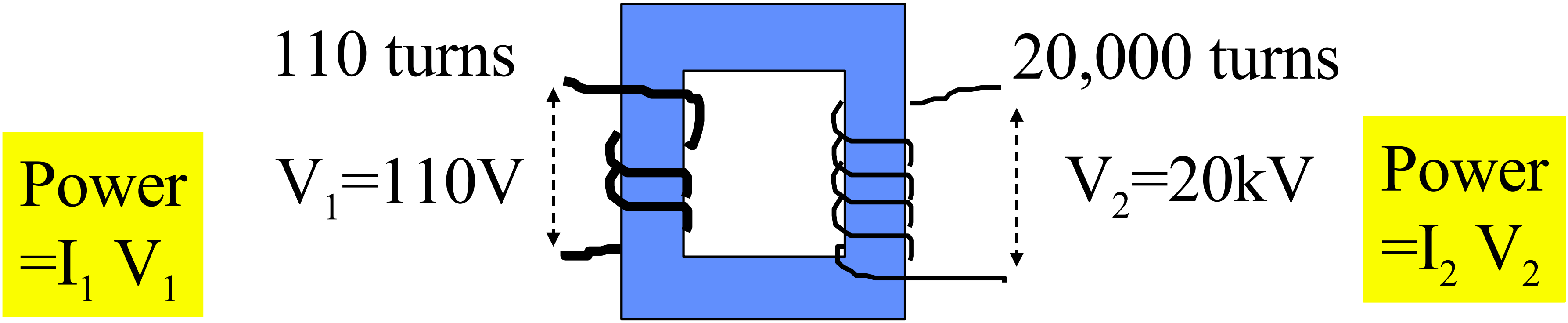
$$V_2 = \frac{N_2}{N_1} V_1$$



Power is conserved, though: $I_1 V_1 = I_2 V_2$
(if 100% efficient.)

Transformers & Power Transmission

Transformers can be used to "step up" and "step down" voltages for power transmission.



We use high voltage (e.g. 365 kV) to transmit electrical power over long distances.

Why do we want to do this?

Transformers & Power Transmission

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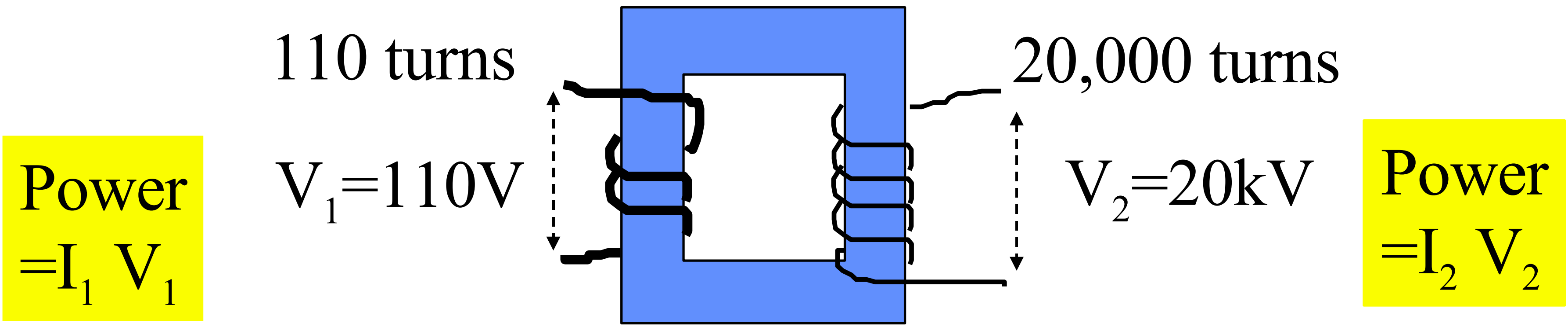

We use high voltage (e.g. 365 kV) to transmit electrical power over long distances.

Why do we want to do this? $P = I^2R$

(P = power dissipation in the line - I is smaller at high voltages)