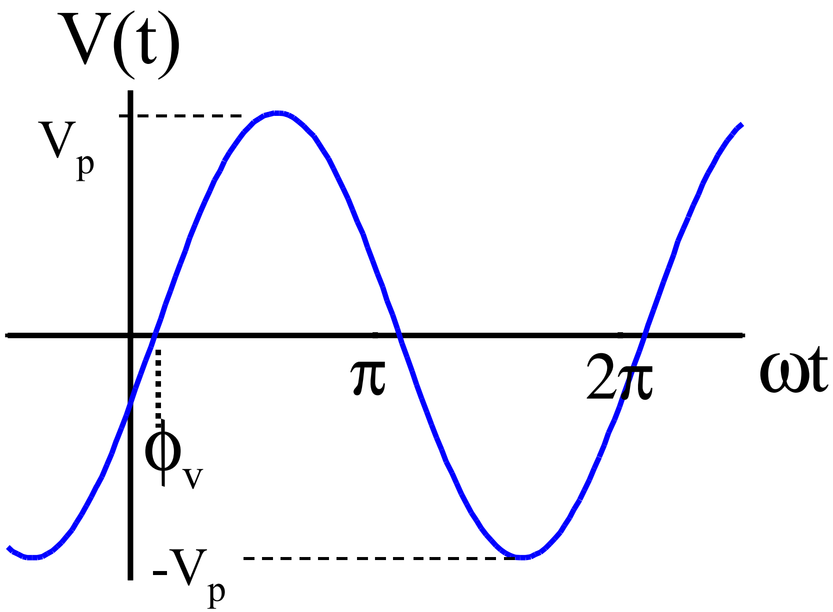
Alternating Current Circuits

Chapter 33

Note: This topic requires you to understand and manipulate sinusoidal quantities which include phase differences. It is a good idea to review these topics in Chapters 15 and 16.

Alternating Current Circuits

An "AC" circuit is one in which the driving voltage and hence the current are sinusoidal in time.



$$V = V_P \sin(\omega t - \phi_v)$$
$$I = I_P \sin(\omega t - \phi_I)$$

ω is the **angular frequency** (angular speed) [radians per second].

Sometimes instead of ω we use the **frequency** f [cycles per second]

Frequency $\equiv$ **f** [cycles per second, or Hertz (Hz)] $\omega = 2\pi f$

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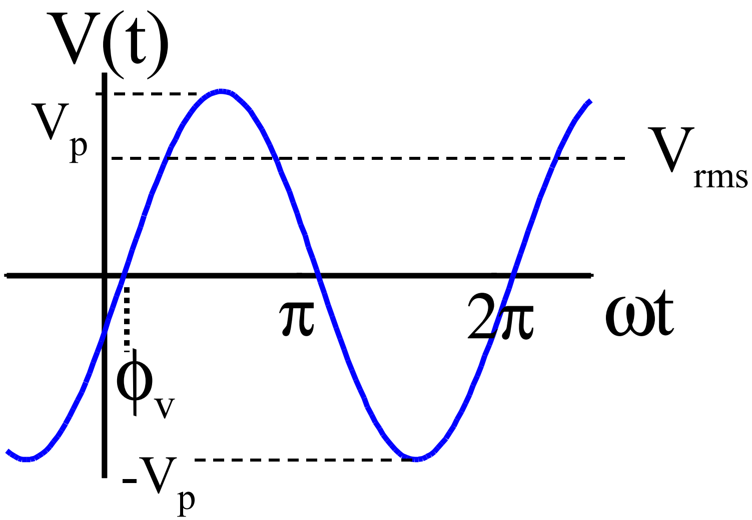

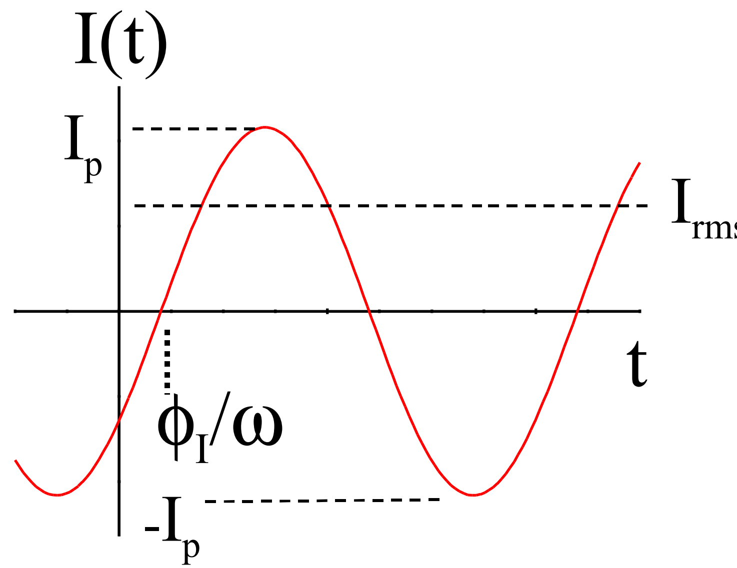

V_p and I_p are the peak current and voltage. We also use the "root-mean-square" values: $V_{rms} = V_p/\sqrt{2}$ and $I_{rms}=I_p/\sqrt{2}$

ϕ_v and ϕ_I are called phase differences (these determine when V and I are zero). Usually we're free to set ϕ_v=0 (but not ϕ_I).

Example: household voltage

In the U.S., standard wiring supplies 120 V at 60 Hz. Write this in sinusoidal form, assuming V(t)=0 at t=0.

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This 60 Hz is the frequency f: so $\omega = 2\pi f = 377\ s^{-1}$.

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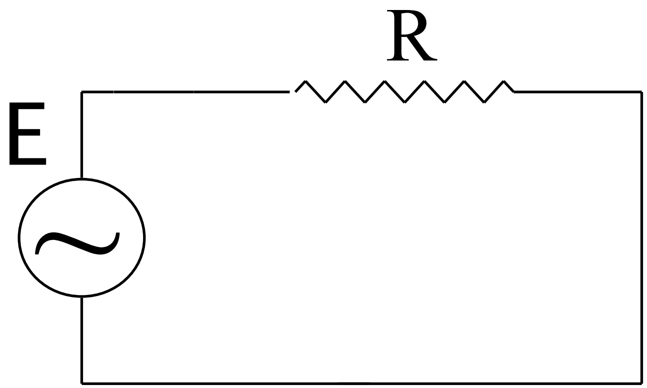
Write this in sinusoidal form, assuming V(t)=0 at t=0.

This 120 V is the RMS amplitude: so $V_p = V_{rms}\sqrt{2} = 170$ V.
This 60 Hz is the frequency f: so $\omega = 2\pi f = 377\ s^{-1}$.

So $V(t) = 170 \sin(377t + \phi_v)$.
Choose $\phi_v = 0$ so that V(t)=0 at t=0: $V(t) = 170 \sin(377t)$.

Resistors in AC Circuits



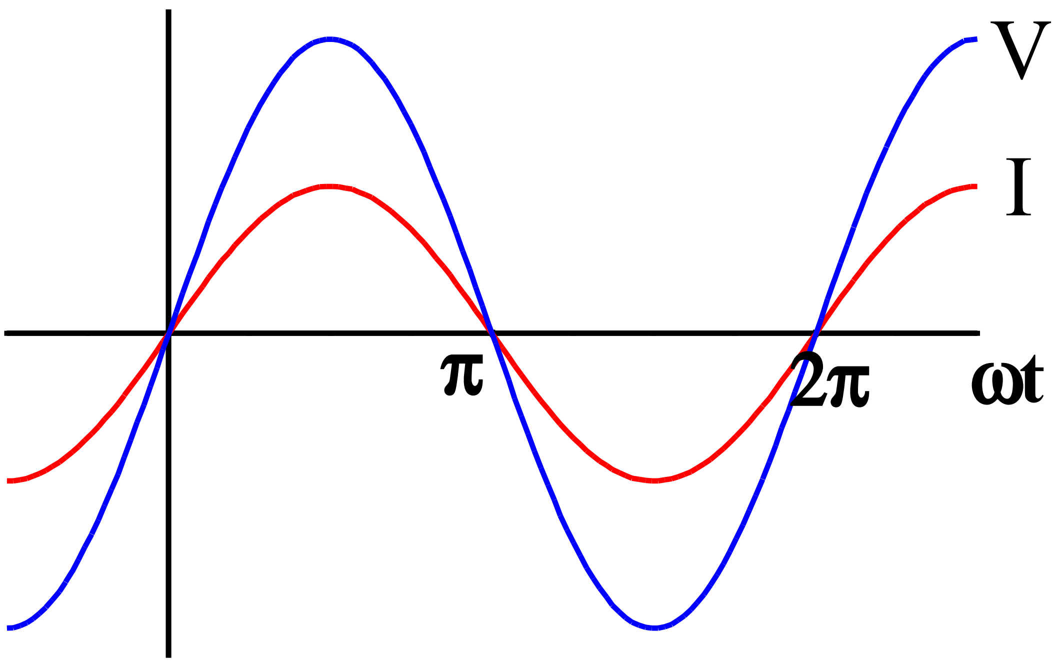
EMF (and also voltage across resistor):

$$V = V_P \sin(\omega t)$$

Hence by Ohm's law, I=V/R:

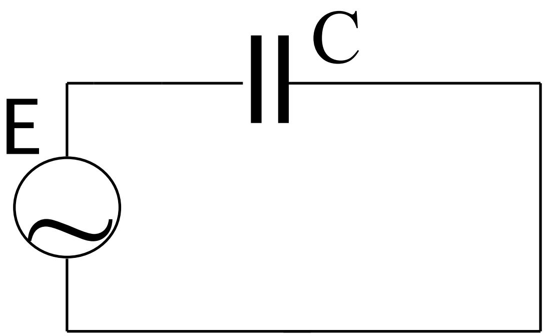
$$I = (V_P / R) \sin(\omega t) = I_P \sin(\omega t)$$

(with $I_P = V_P/R$)



V and I
"In-phase"

Capacitors in AC Circuits

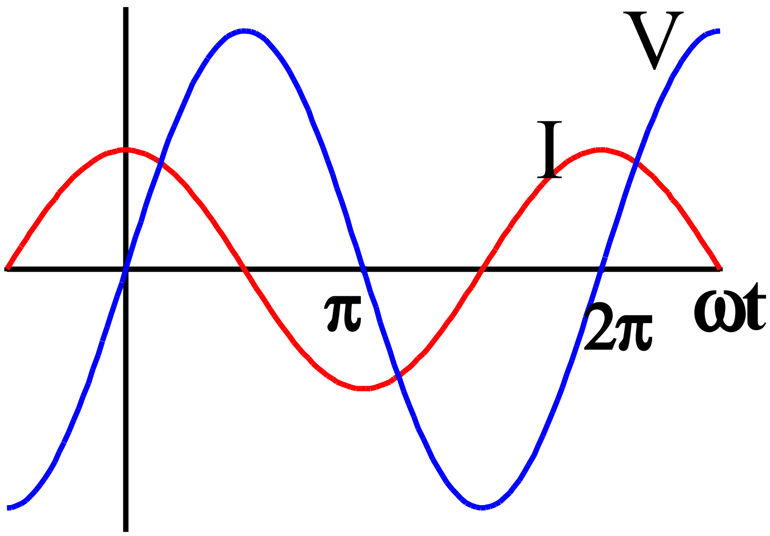


Start from: $q = C V$ $[V = V_p \sin(\omega t)]$

Take derivative: $dq/dt = C\, dV/dt$

So $I = C\, dV/dt = C V_p\, \omega \cos(\omega t)$

$$I = C \omega\, V_P \sin(\omega t + \pi/2)$$



This looks like $I_P = V_P/R$ for a resistor (except for the phase change).

So we call $X_c = 1/(\omega C)$

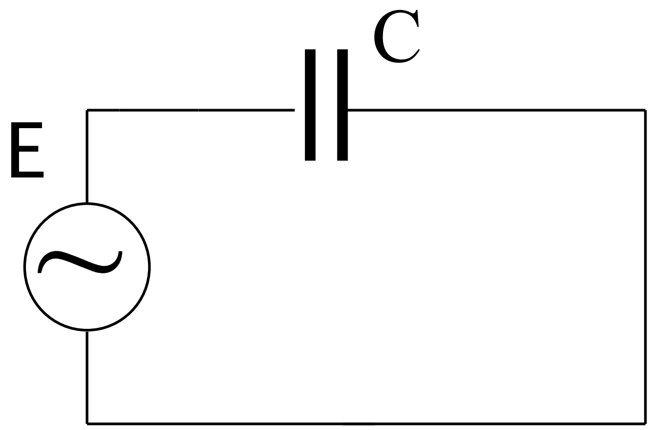
the Capacitive Reactance

The reactance is sort of like resistance in that $\mathbf{I_P = V_P/X_c}$. Also, the current leads the voltage by 90° (phase difference).

V and I "out of phase" by 90°. I leads V by 90°.

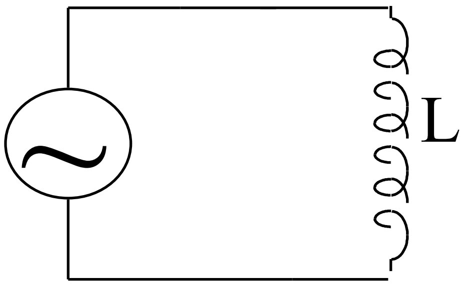
Capacitor Example

A 100 nF capacitor is connected to an AC supply of peak voltage 170V and frequency 60 Hz.



What is the peak current?
What is the phase of the current?
What is the dissipated power?

Inductors in AC Circuits



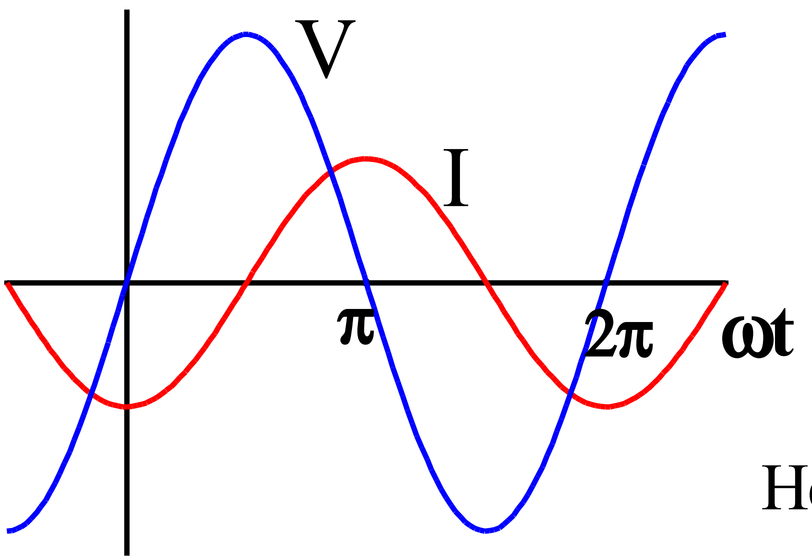
$V = V_P \sin(\omega t)$

Loop law: $V + V_L = 0$ where $V_L = -L\, dI/dt$

Hence: $dI/dt = (V_P/L) \sin(\omega t)$.

Integrate: $I = -(V_P / L\omega) \cos(\omega t)$

or $I = [V_P /(\omega L)] \sin(\omega t - \pi/2)$



Again this looks like $I_P = V_P/R$ for a resistor (except for the phase change).

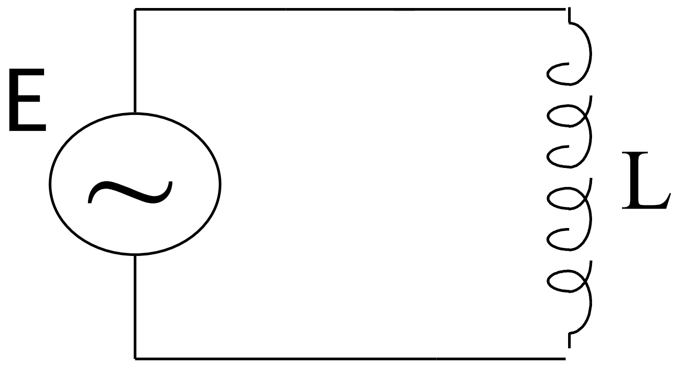
So we call the $X_L = \omega L$ Inductive Reactance

Here the current lags the voltage by 90°.

V and I "out of phase" by 90°. I lags V by 90°.

Inductor Example

A 10 mH inductor is connected to an AC supply of peak voltage 10V and frequency 50 kHz.



What is the peak current?
What is the phase of the current?
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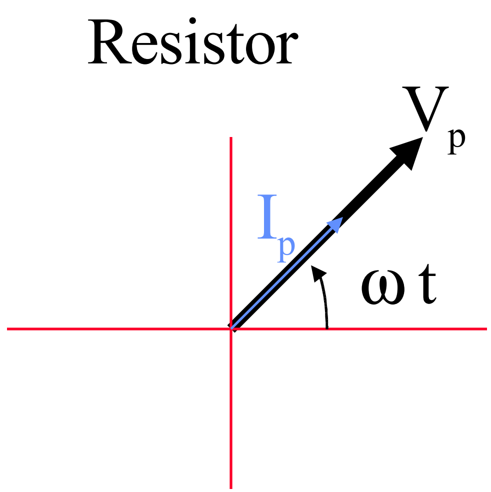
CIRCUIT ELEMENT	REACTANCE	AMPLITUDE RELATION	PHASE RELATION
resistor	R	$I_0 = V_0/R$	I, V in phase
capacitor	$X_C = 1/\omega C$	$I_0 = V_0/X_C$	I leads V by 90°
inductor	$X_L = \omega L$	$I_0 = V_0/X_L$	V leads I by 90°

Phasor Diagrams

A phasor is an arrow whose length represents the amplitude of an AC voltage or current.
The phasor rotates counterclockwise about the origin with the angular frequency of the AC quantity.
Phasor diagrams are useful in solving complex AC circuits.
The "y component" is the actual voltage or current.



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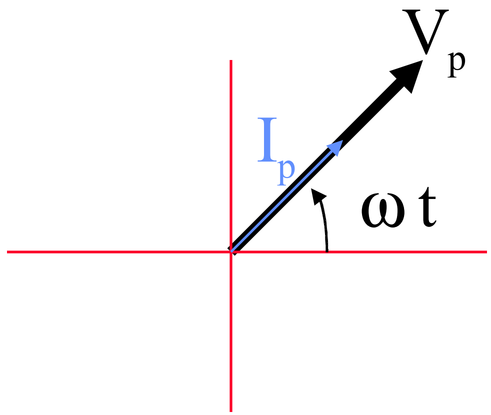

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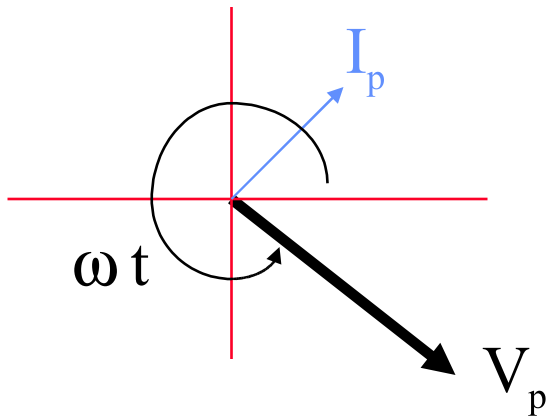

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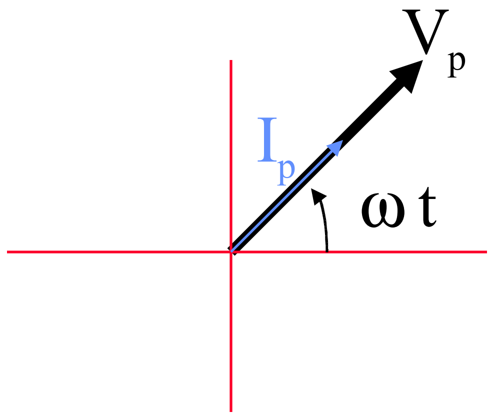

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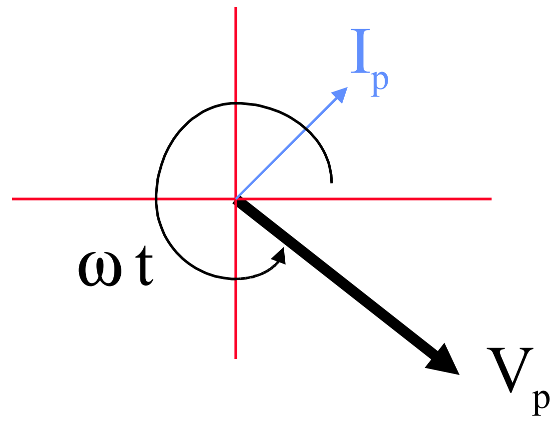
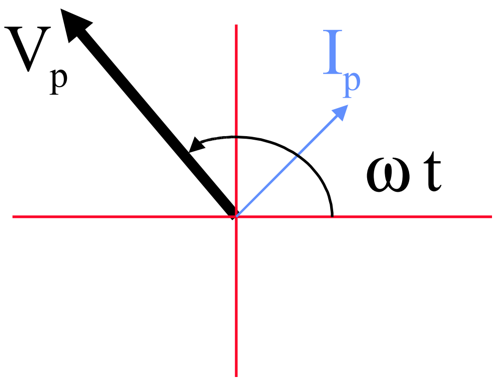
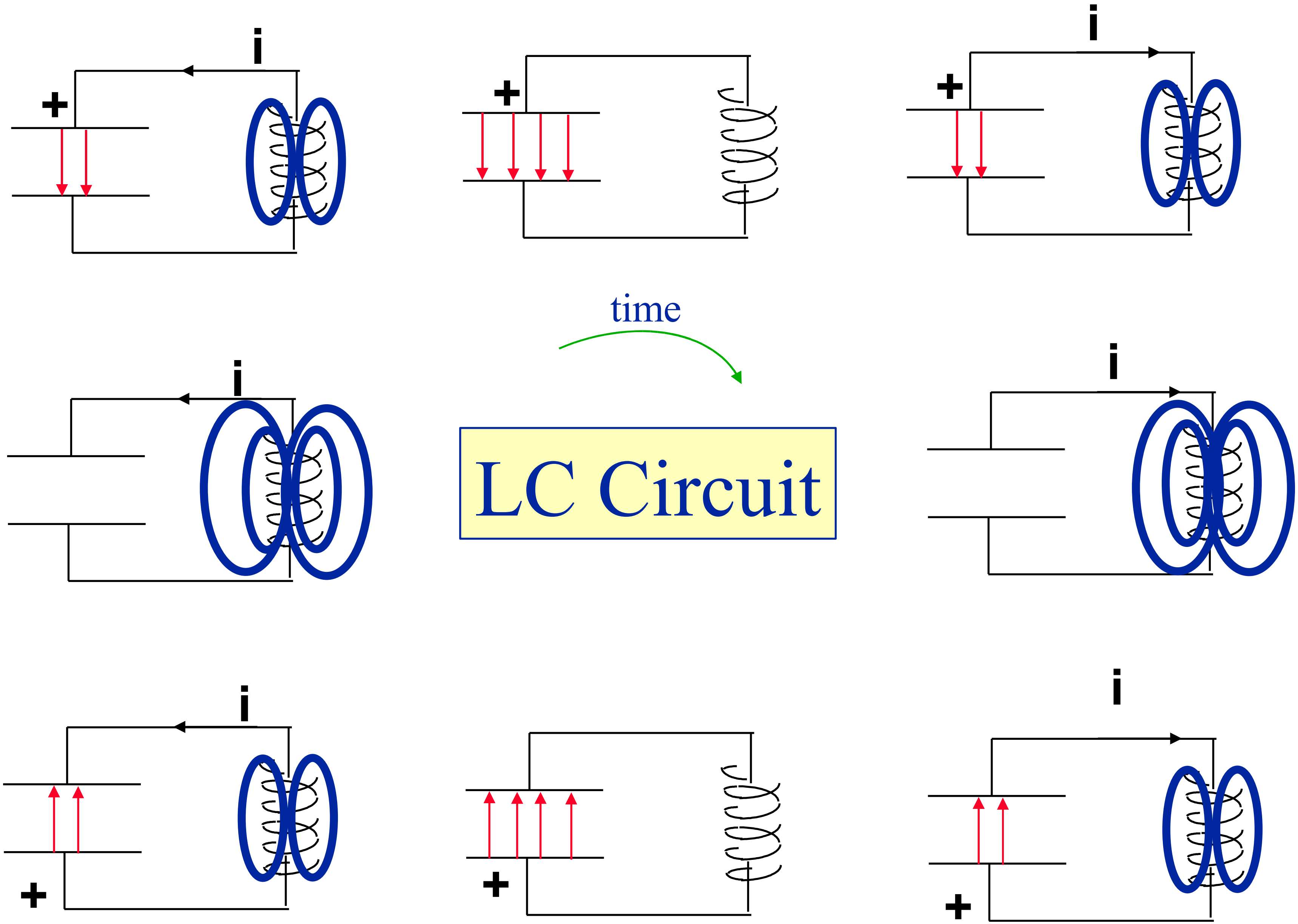


Inductor




LC Circuit
time
i
+

Analyzing the L-C Circuit

Total energy in the circuit: $U = U_B + U_E = \frac{1}{2}LI^2 + \frac{1}{2}\frac{q^2}{C}$

Differentiate : $\frac{dU}{dt} = \frac{d}{dt}\left(\frac{1}{2}LI^2 + \frac{1}{2}\frac{q^2}{C}\right) = 0$ **N o change in energy**

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$$\frac{d^2q}{dt^2} + \omega^2 q = 0$$

$$q = q_p \cos\omega t$$

The charge sloshes back and forth with frequency $\omega = (LC)^{-1/2}$