# Inductance and Magnetic Energy

## Chapter 32

Mutual Inductance

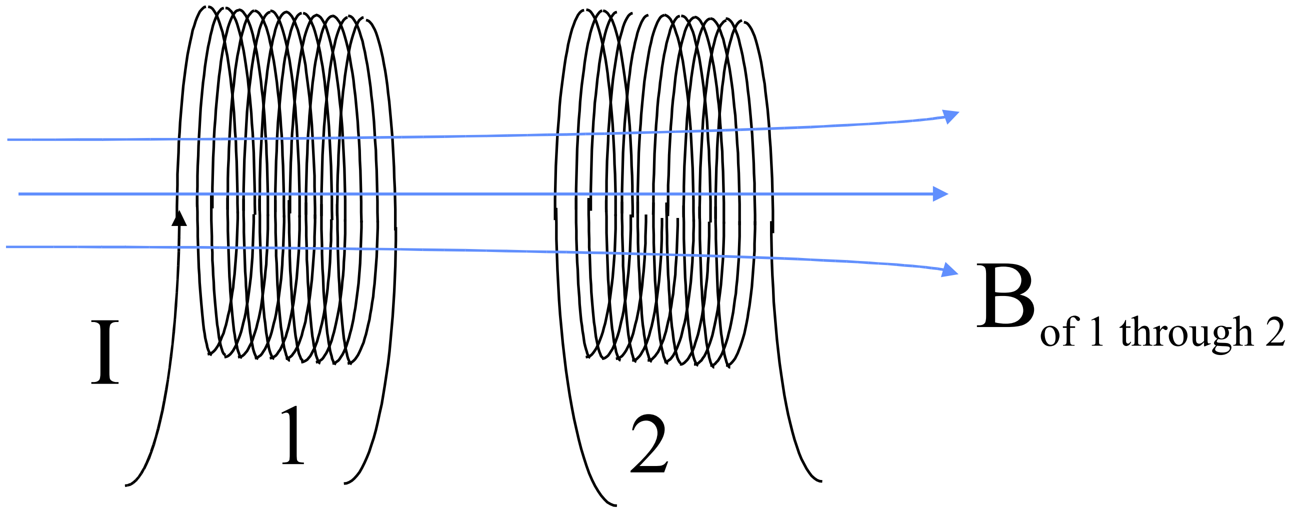
Self-Inductance

Inductors in Circuits

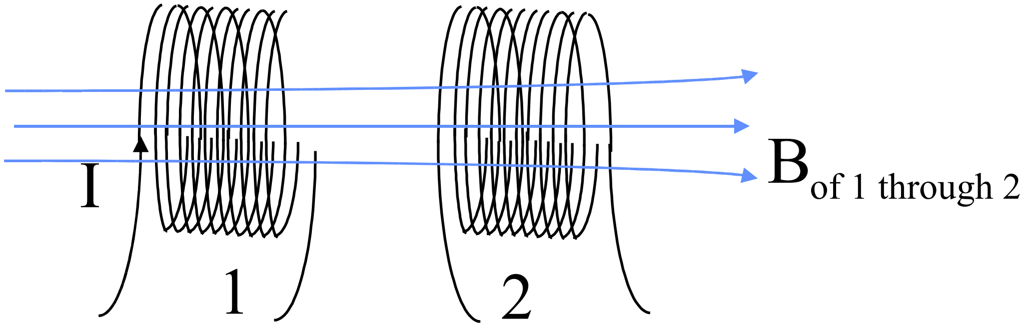
Magnetic Energy

# Mutual Inductance

- Two coils, 1 & 2, are arranged such that flux from one passes through the other.
- We already know that changing the current in 1 changes the flux (in the other) and so induces an emf in 2.
- This is mutual inductance.



# Definition of the Mutual Inductance



The *mutual inductance*, M, tells us how much flux through the second coil, $\Phi_2$ , is caused by a current, $I_1$, through the first:

$M = \Phi_2/I_1$ which gives $\Phi_2 = M\,I_1$

so: $d\Phi_2/dt = M\,dI_1/dt$

But by Faraday's law :

$$E_2 = -\,d\,\Phi_2/dt = -\,M\,dI_1/dt$$

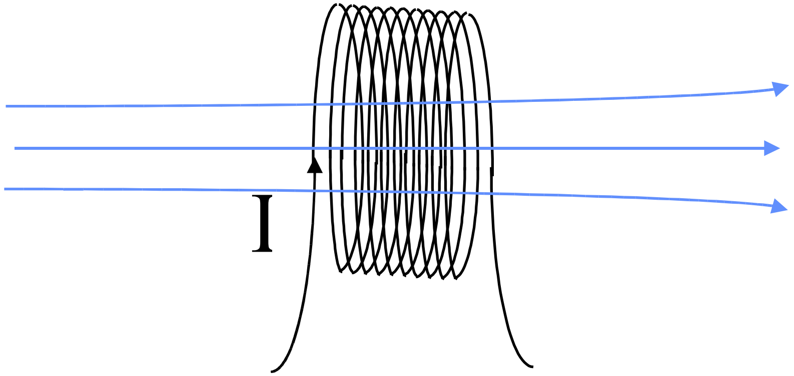
# Mutual Inductance is Geometric

- M arises from the way flux from one coil passes through the other: that is from the geometry and arrangement of the coils.
- Mutual means mutual. Note there is no subscript on M: the effect of 2 on 1 is identical to the effect of 1 on 2.
- Inductance has units: called the "Henry" (H).

$$1\ \text{H} = 1\ \text{Vs/A}$$

# Self Inductance

*A changing current in a coil can induce an emf* *in itself*….



- If the current is steady: no problem, the coil acts like an ordinary piece of wire.
- But if the current changes, B changes and so then does $\Phi_B$, and Faraday tells us there will be an induced emf.
- Lenz’s law tells us that the induced emf must be in such a direction as to produce a current which makes a magnetic field opposing the change.

# Implications of Self Inductance

- Define the self inductance of a circuit element (a coil, wire, resistor or whatever) as

  $L = \Phi_B / I$

- From this we have $\Phi_B = LI$ and so

  $d\,\Phi_B/dt = LdI/dt$

- and Faraday's law gives

  $\mathcal{E} = -\,L\,dI/dt$

- Since this emf opposes changes in the current (in the component) it is often called the "back emf".
- From now own "inductance" means self-inductance.

# Example: Finding Inductance

What is the (self) inductance of a solenoid ($L = \Phi_B/I$) with area A, length d, and n turns per unit length?

In the solenoid $B = \mu_0 nI$, so the flux through one turn is: $\phi_B = BA = \mu_0 nIA$

The total flux in the solenoid is $(nd)\phi_B$

Therefore, $\Phi_B = \mu_0 n^2 IAd$ and so $L = \Phi_B/I$ gives

$$\boxed{L = \mu_0 n^2 Ad}$$

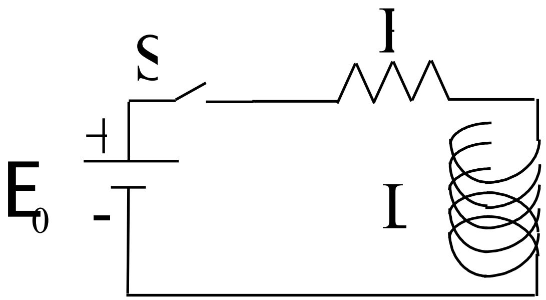
(only geometry)

# Inductance Affects Circuits and Stores Energy

- This has important implications…..
- First an observation: Since $E$ cannot be infinite neither can dI/dt. Therefore, current *cannot change instantaneously*.
- We will see that inductance in a circuit affects current in somewhat the same way that capacitance in a circuit affects voltage.
- A ‘thing’ (a component) with inductance in a circuit is called an inductor.

# Circuits With Inductance

We start with a simple circuit containing a battery, a switch, a resistor, and an inductor, and follow what happens when the switch is closed.



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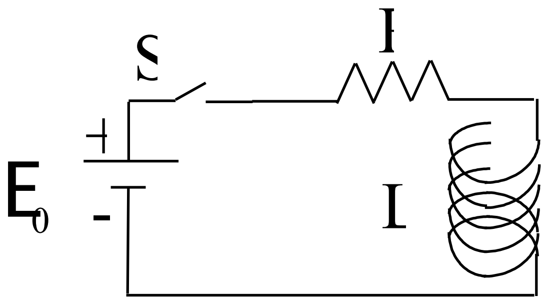

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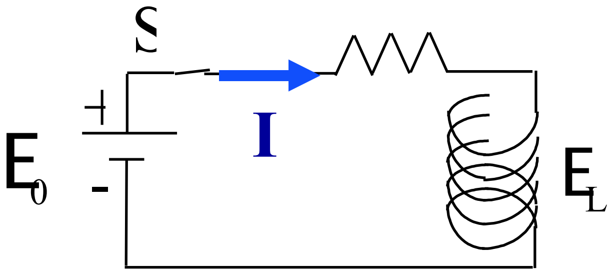

When the switch is closed current I flows, growing gradually, and a ‘back emf’ $E_L$ is generated in inductor.

The emf $E_L$ opposes the current I

$\Rightarrow$ $E_L = - L\, dI/dt$

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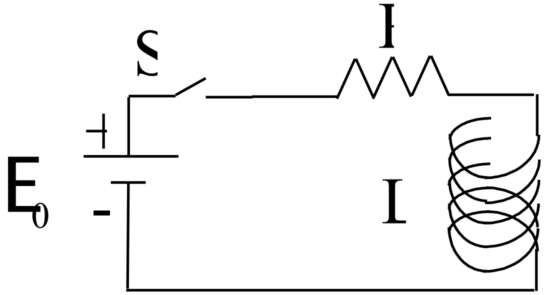

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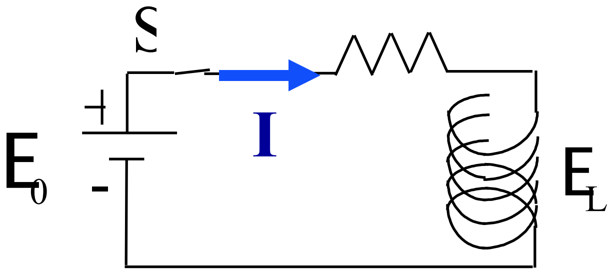

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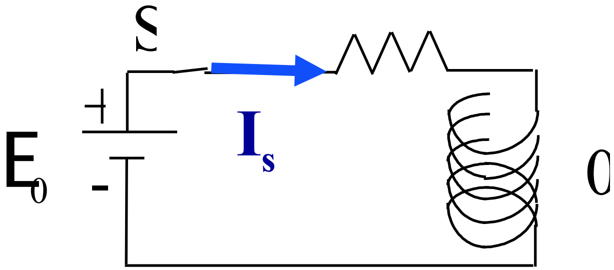
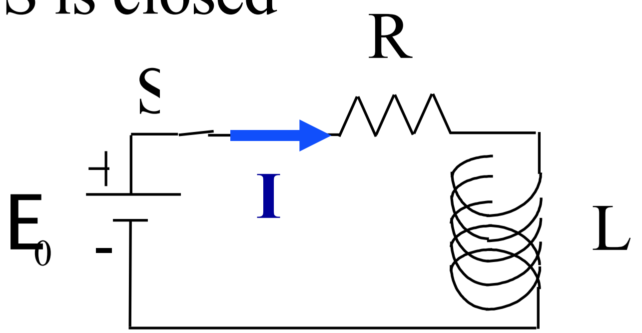


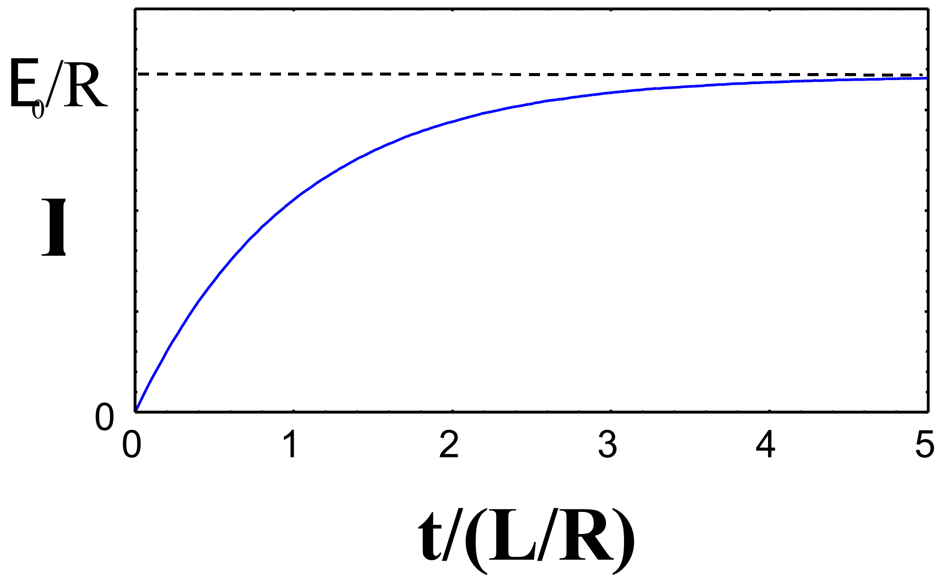
After a long time the current becomes steady. Then $E_L$ is zero.

# Circuits With Inductance

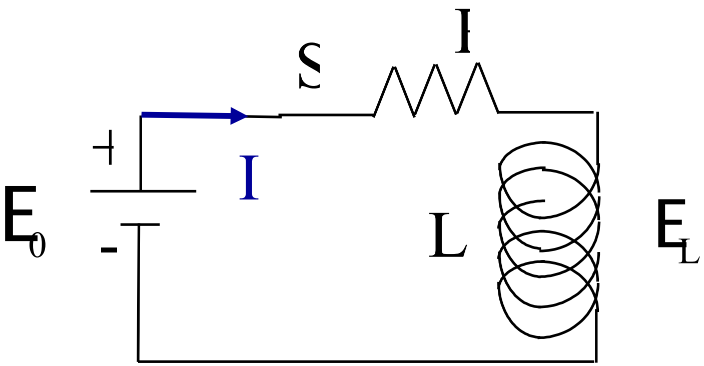
When the switch S is closed



The current **I** increases exponentially from **I** = 0 to **I** = $E_0/R$



# Analysis of the Establishment of a Current



We use the loop method

$$E_0 - IR + E_L = 0 \Rightarrow E_0 - IR - L\,dI/dt = 0$$

$$IR = E_0 - L\,dI/dt \Rightarrow R(I-(E_0/R)) = - L\,dI/dt$$

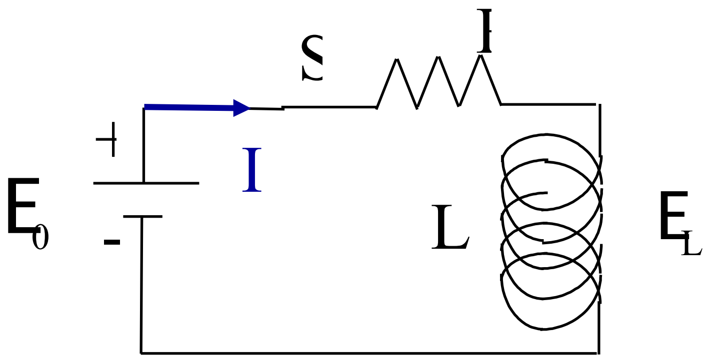
$$dI / (I-(E_0/R)) = - dt / (L/R) \Rightarrow \int dI / (I-(E_0/R)) = - \int dt / (L/R)$$

$$\ln (I-(E_0/R)) - \ln -(E_0/R) = - t / (L/R) \Rightarrow \ln (-I/(E_0/R) + 1) = - t / (L/R)$$

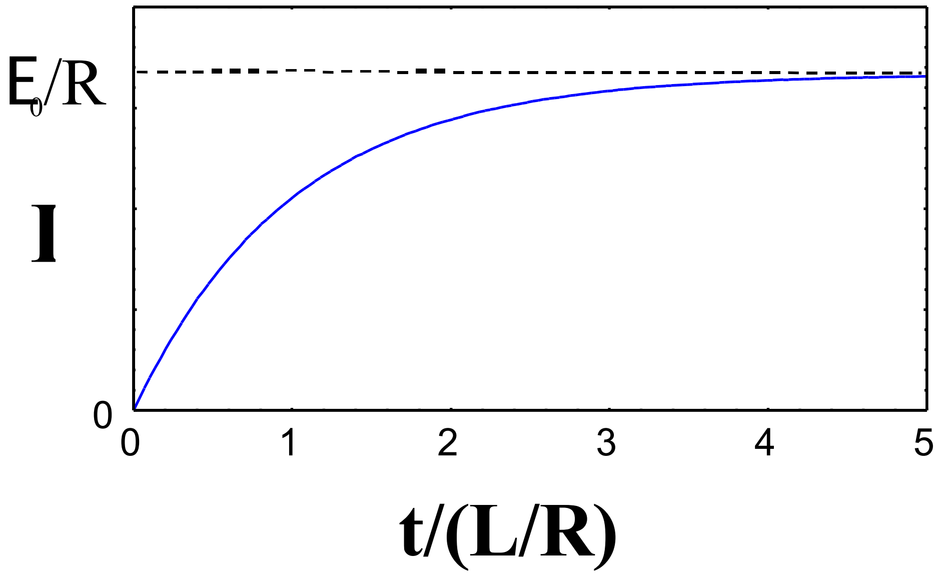
$$-I/(E_0/R) + 1 = \exp (- t / (L/R)) \Rightarrow I/(E_0/R) = 1 - \exp (- t / (L/R))$$

$$\boxed{I = (E_0/R) [1 - \exp (- t / (L/R))]}$$

# Analysis of the Establishment of a Current



$$I = (E_0/R)\,[1 - \exp(-t/(L/R))]$$

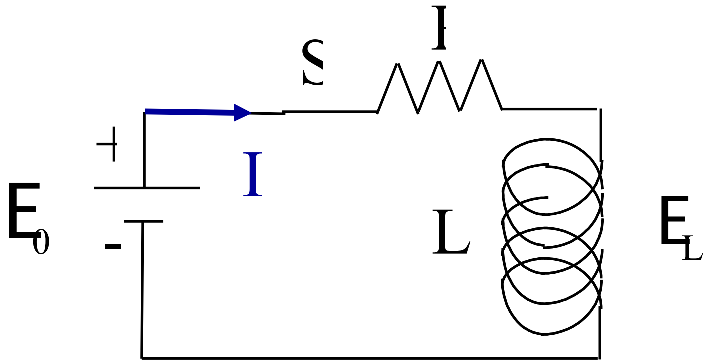


The current increases
exponentially
with time constant
$\tau = L / R$

$t = 0 \Rightarrow I = 0$

$t = \infty \Rightarrow I = E_0/R$

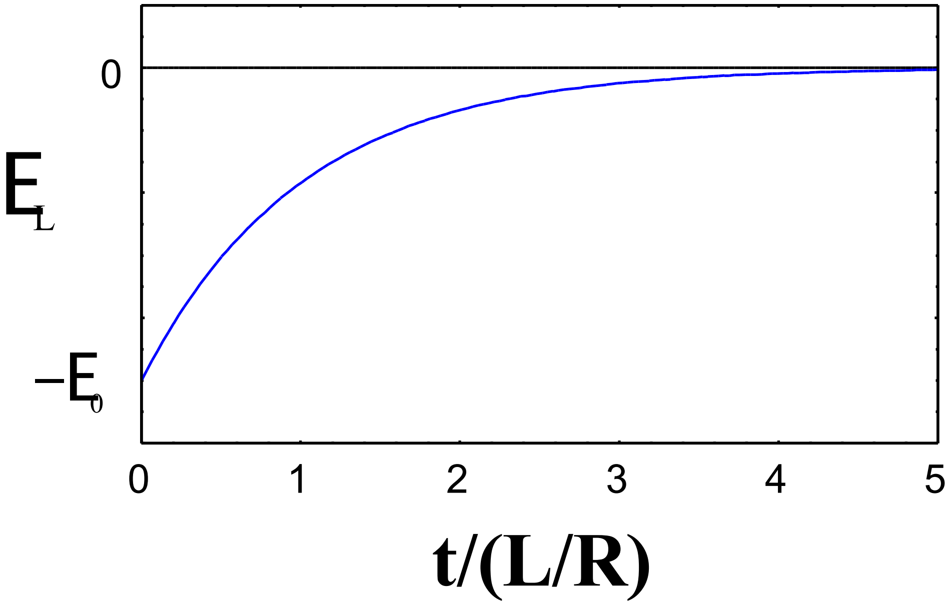
# Inductor's emf $E_L$



$$E_L = - L\ (dI/dt)$$

$$I = (E_0/R)\ [1 - \exp(- t / (L/R))]$$

$$E_L = L\ (E_0/R)\ (-R/L)\ \exp(- t / (L/R))$$

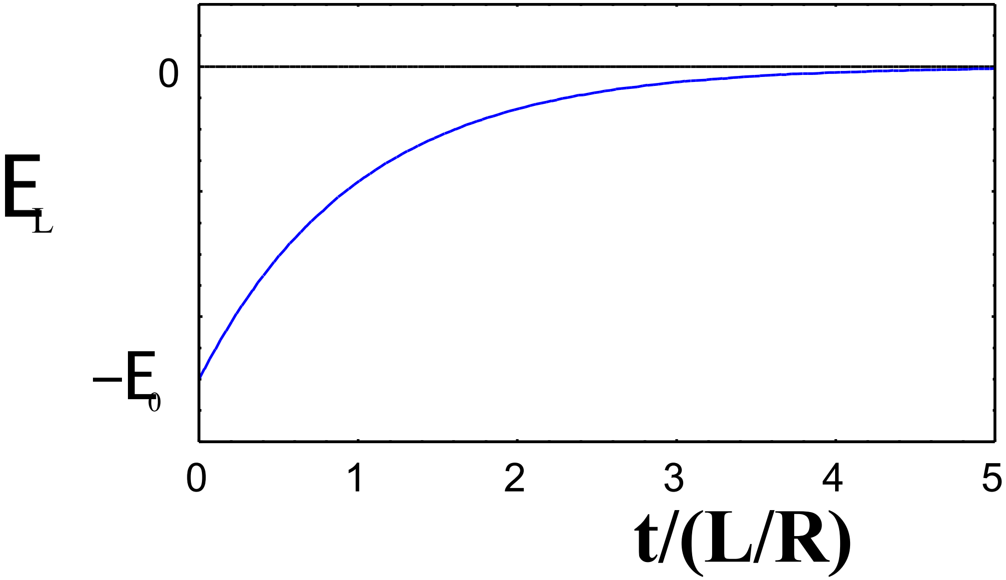


$$\boxed{E_L = - E_0 \exp(- t / (L/R))}$$

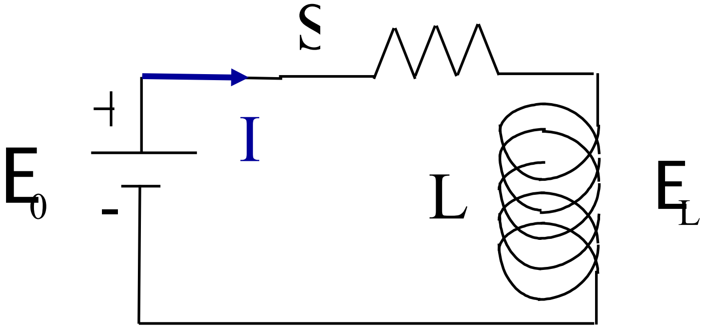
$t = 0 \Rightarrow E_L = - E_0$

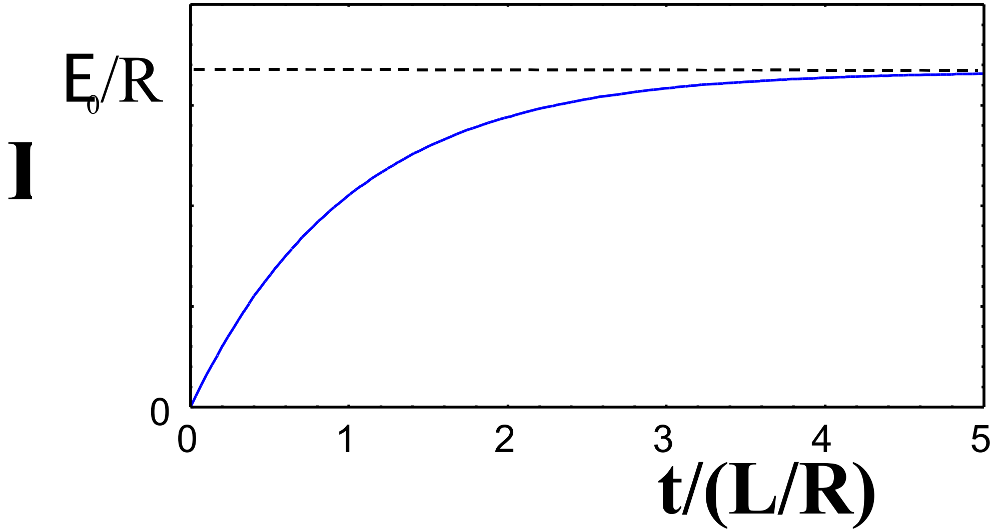
$t = \infty \Rightarrow E_L = 0$

# Plots for the “RL” Circuit



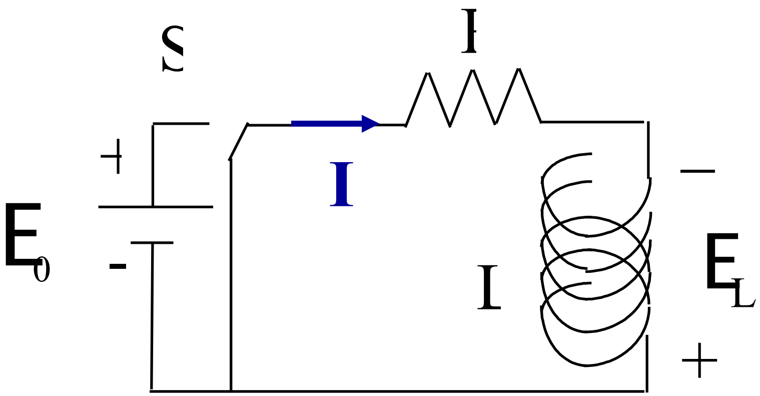
$$E_L = - E_0 \exp(- t / (L/R))$$





$$I = (E_0/R)\,[1 - \exp(- t / (L/R))]$$

# Decay of an “RL” Circuit



- After I reaches $E_0/R$ move the switch as shown.
- The loop method gives $E_L - IR = 0$ or $E_L = IR$
- Remember that $E_L = -L\, dI/dt \Rightarrow -L\, dI/dt = IR$

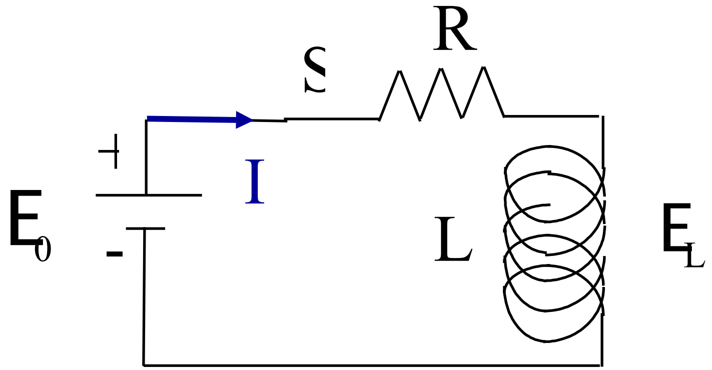
  $dI/I = -\, dt / (L/R) \Rightarrow \int dI/I = -\int dt / (L/R)$
- $\ln I_0/I = -\, t / (L/R) \Rightarrow I = I_0 \exp[-\, t / (L/R)]$
- But $I_0 = E_0/R$
- Then: $I = (E_0/R) \exp[-\, t / (L/R)]$

# Inductors in Circuits

- The presence of inductance prevents currents from changing instantaneously.
- The time constant of an “RL” circuit is $\tau = L/R$.
- Next we will see that inductors store energy because they confine magnetic fields.

  (This is very similar to the idea that capacitors store energy in the confined electric fields.)

# Energy Stored in an Inductor



Recall the original circuit when current was changing (building up). The loop method gave: $E_0 - IR + E_L = 0$

Multiply by I, and use Faraday's law for $E_L$ ($E_L = - L\, dI/dt$)

Then:

$$IE_0 - I^2R - ILdI/dt = 0$$

or:

$$IE_0 - I^2R - d[(1/2)LI^2]/dt = 0 \quad \{d[(1/2)LI^2]/dt = ILdI/dt\}$$

- Think about $IE_0 - I^2R - d((1/2)LI^2)/dt = 0$
- $IE_0$ is the power (energy per unit time) delivered by the battery.
- $I^2R$ is the power dissipated in the resistor.

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  ➔In creating the magnetic field in the inductor
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- The amount of energy in the magnetic field is:

$$\boxed{\mathbf{U_B = \tfrac{1}{2} LI^2}}$$

# Energy Density in a Magnetic Field

- We have shown $U_B = \frac{1}{2} L I^2$ (solenoid).
- Therefore,

$$U_B = \frac{1}{2}\mu_o n^2 A\ell I^2 = \frac{A\ell}{2\mu_o}\left(\mu_o^2 n^2 I^2\right) = \frac{A\ell}{2\mu_o} B^2$$

- Since A **l** is the volume of the solenoid, the stored energy density is:

$$\boxed{u_B = B^2/(2\mu_0)}$$

- This turns out to be the

*energy density in a magnetic field*

# Summary

- We defined mutual and self inductance,
- Calculated the inductance of a solenoid.
- Saw the effect of inductance in “RL” circuits.
- Developed an expression for the stored energy.
- Derived an expression for the energy density of a magnetic field.
- Next class we will start learning about alternating-current (AC) circuits, containing resistors, capacitors, and inductors.