Electromagnetic Induction

Chapter 31

Faraday's Law

Induced currents

Lenz's Law

Induced EMF

Magnetic Flux

Induced Electric fields

Electromagnetic Induction

A changing magnetic field (intensity, movement) will induce an electromotive force (emf)

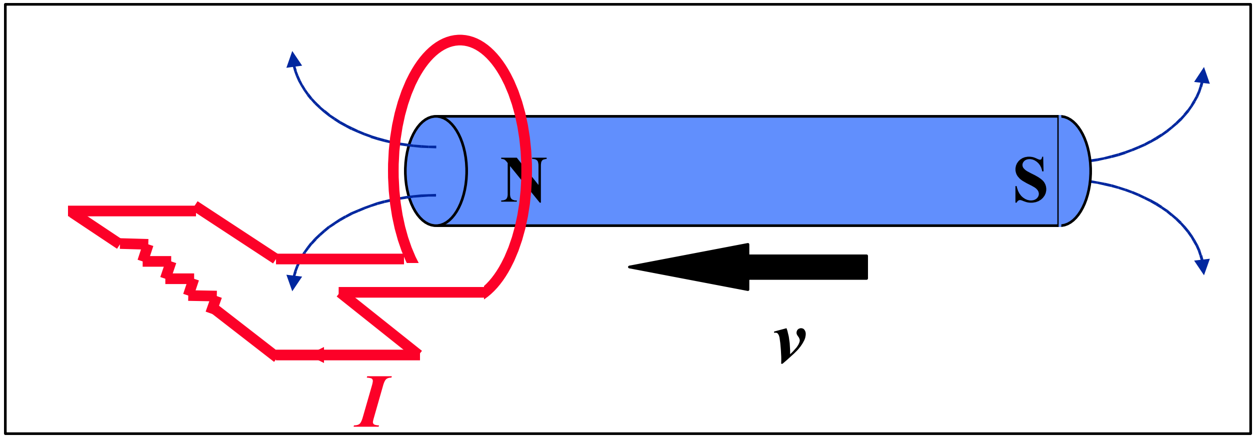
In a closed electric circuit, a changing magnetic field will produce an electric current

Electromagnetic Induction
Faraday's Law

The induced emf in a circuit is proportional to the rate of change of magnetic flux, through any surface bounded by that circuit.

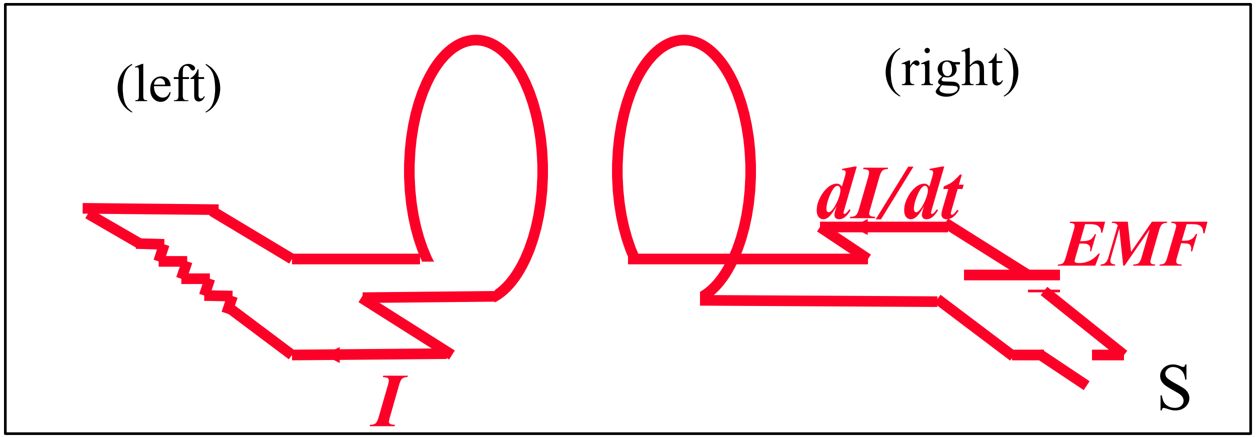
$$\mathcal{E} = -\, d\Phi_B / dt$$

Faraday's Experiments



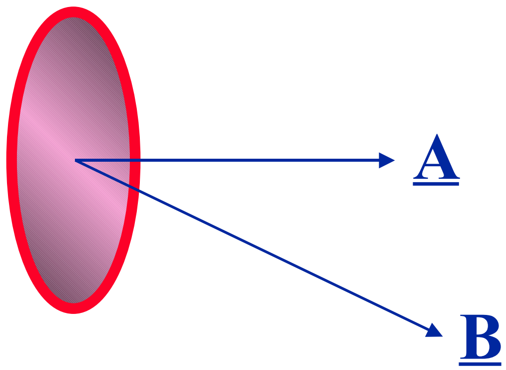
- Michael Faraday discovered induction in 1831.
- Moving the magnet induces a current I.
- Reversing the direction reverses the current.
- Moving the loop induces a current.
- The induced current is set up by an *induced EMF*.

Faraday's Experiments



- Changing the current in the right-hand coil induces a current in the left-hand coil.
- The induced current does not depend on the size of the current in the right-hand coil.
- The induced current depends on ***dI/dt***.

Magnetic Flux

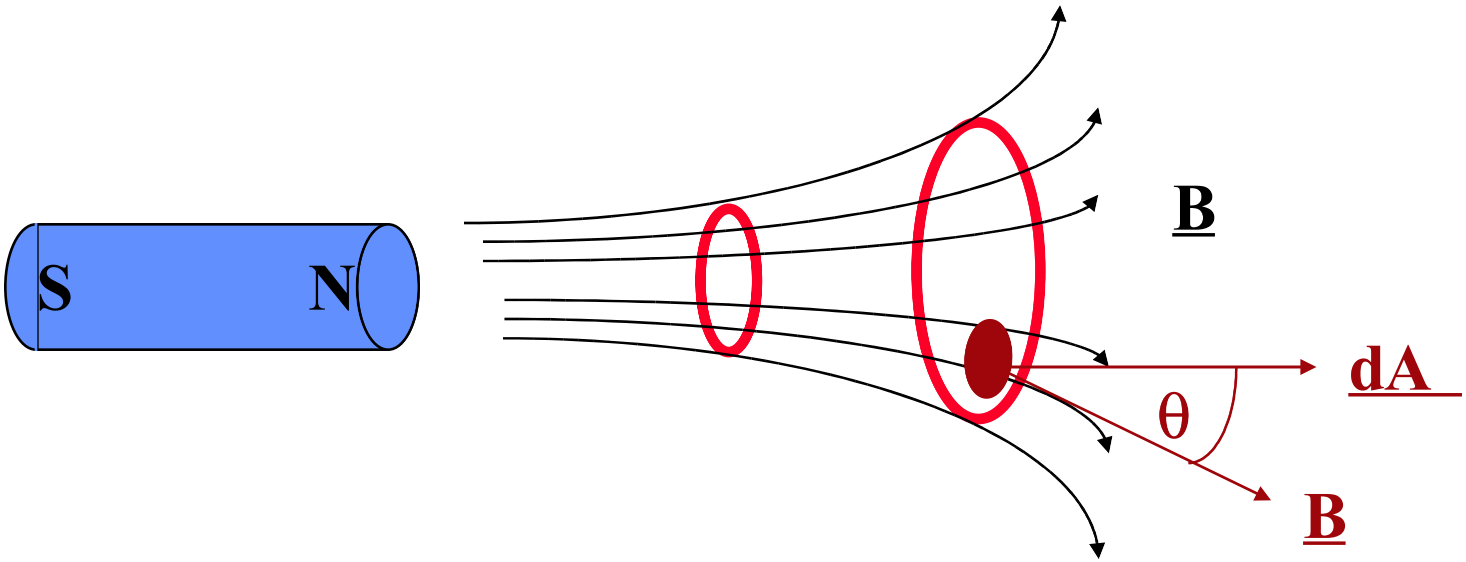


- In the easiest case, with a constant magnetic field **B**, and a flat surface of area **A**, the magnetic flux is

$$\Phi_B = \underline{\mathbf{B}} \cdot \underline{\mathbf{A}}$$

- Units : 1 tesla x m^2 = 1 weber

Magnetic Flux



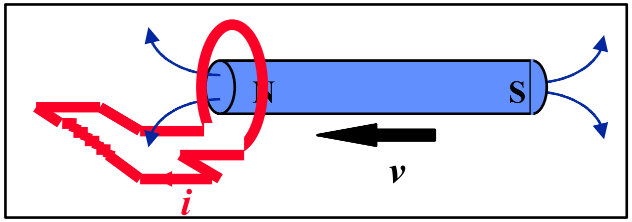
- When B is not constant, or the surface is not flat, one must do an integral.
- Break the surface into bits **dA**. The flux through one bit is

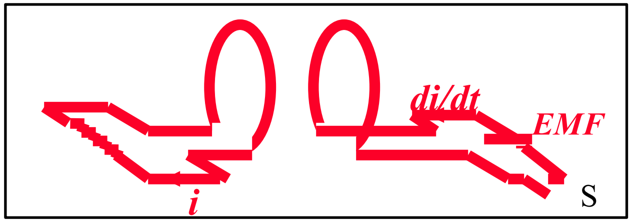
$$d\Phi_B = \mathbf{B} \cdot d\mathbf{A} = B\, dA \cos\theta.$$

- Add the bits:

$$\Phi_B = \int B \cdot dA = \int B \cos\theta \, dA$$

Faraday's Law

1) 

2) 

- Moving the magnet changes the flux Φ_B (1).
- Changing the current changes the flux Φ_B (2).
- ***Faraday:*** changing the flux induces an emf.

$$\mathcal{E} = -\, d\Phi_B / dt$$

Faraday's law

The emf induced around a loop

equals the rate of change of the flux through that loop

Lenz's Law

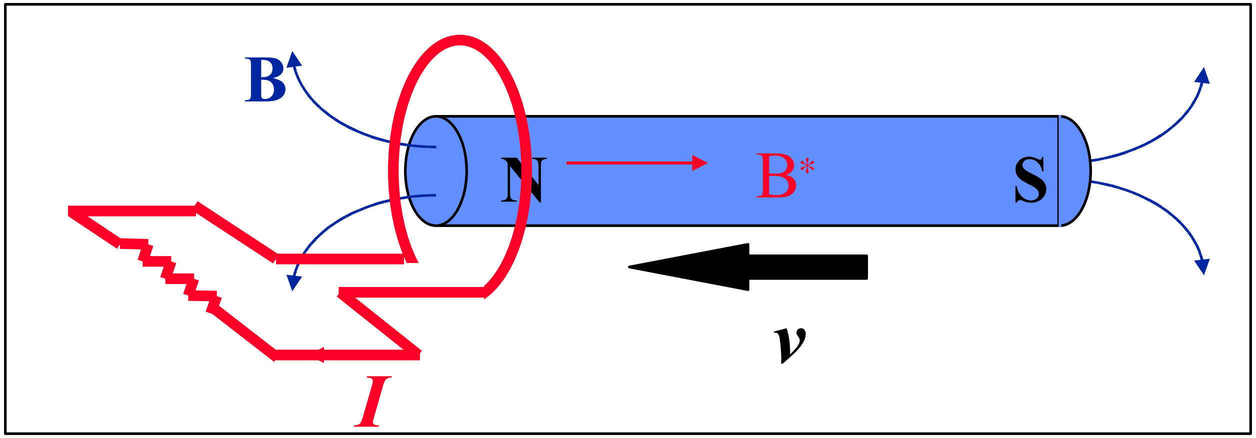
- Faraday's law gives the direction of the induced emf and therefore the direction of any induced current.
- Lenz's law is a simple way to get the directions straight, with less effort.
- **Lenz's Law:**

  The induced emf is directed so that any induced current flow will *oppose* the *change* in magnetic flux (which causes the induced emf).
- This is easier to use than to say ...

  Decreasing magnetic flux ⇒ emf creates additional magnetic field

  Increasing flux ⇒ emf creates opposed magnetic field

Lenz's Law

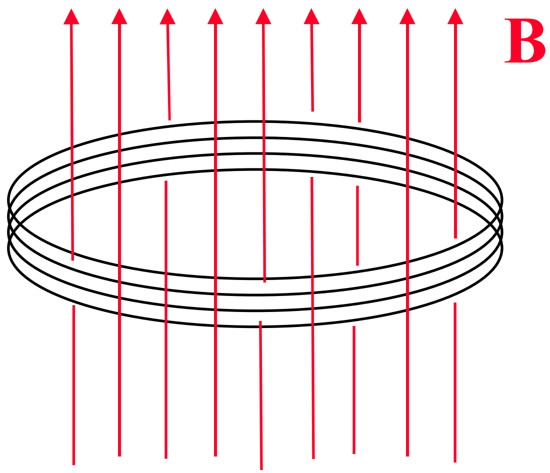


If we move the magnet towards the loop the flux of B will increase.

Lenz's Law ⇒ the current induced in the loop will generate a field B^* opposed to B.

Example of Faraday's Law

Consider a coil of radius 5 cm with N = 250 turns.
A magnetic field B, passing through it,
changes at the rate of dB/dt = 0.6 T/s.
The total resistance of the coil is 8 Ω.
What is the induced current ?

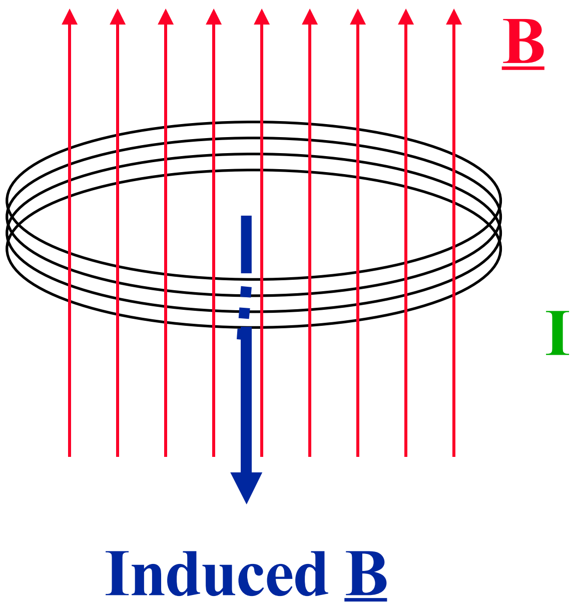


B = 0.6 t [T] (t = time in seconds)

Use Lenz's law to determine the direction of the induced current.

Apply Faraday's law to find the emf and then the current.

Example of Faraday's Law



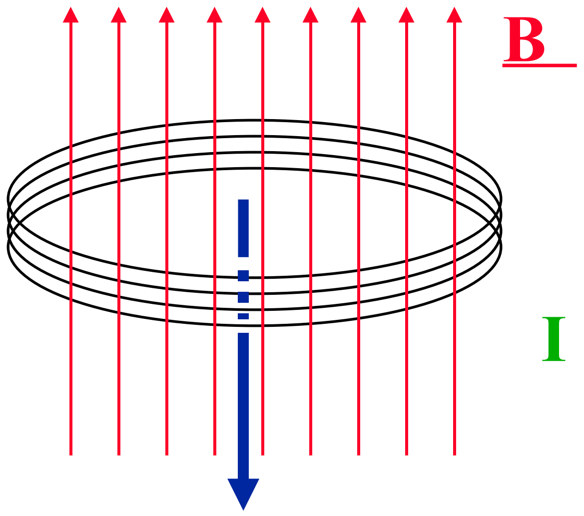
Lenz's law: dB/dt > 0

The change in B is increasing the upward flux through the coil.

So the induced current will have a magnetic field whose flux (and therefore field) is ***down***.

Hence the induced current must be ***clockwise*** when looked at from above.

Use Faraday's law to get the magnitude of the induced emf and current.



$$\Phi_B = \int \underline{B} \, . \, d\underline{A}$$

$$\mathcal{E} = - d\Phi_B /dt$$

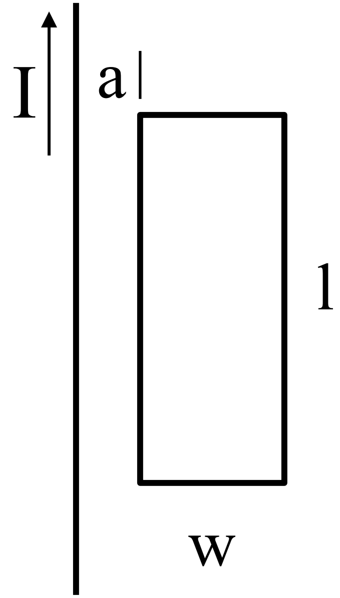
The induced EMF is $\mathcal{E} = - d\Phi_B /dt$

In terms of B: $\Phi_B = N(BA) = NB(\pi r^2)$

Therefore $\mathcal{E} = - N(\pi r^2) \, dB/dt$

$\mathcal{E} = - (250)(\pi \, 0.005^2)(0.6 T/s) = -1.18 \text{ V}$ (1V=1Tm²/s)

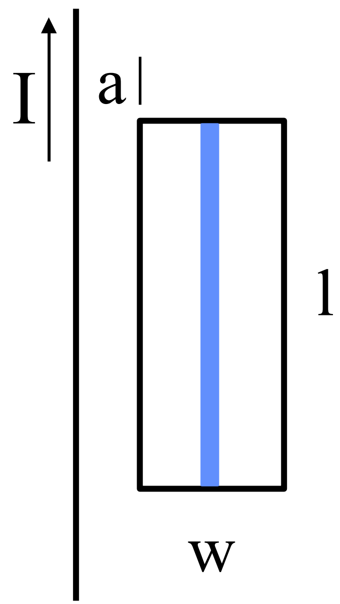
Current I $= \mathcal{E}/R = (-1.18V)/(8\,\Omega) = -0.147$ A

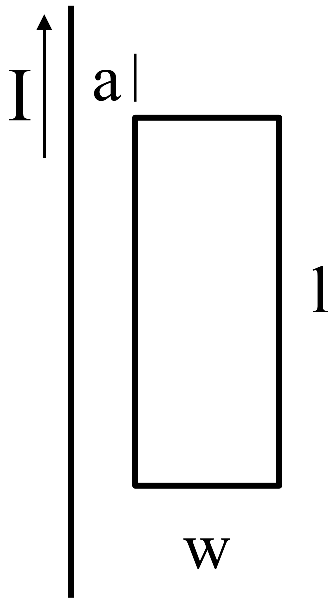


Magnetic Flux in a Nonuniform Field

A long, straight wire carries a current I. A rectangular loop (w by l) lies at a distance a, as shown in the figure.

What is the magnetic flux through the loop?.





Induced emf Due to Changing Current

A long, straight wire carries a current $I = I_0 + i\,t$.

A rectangular loop (w by l) lies at a distance a, as shown in the figure.

What is the induced emf in the loop?.

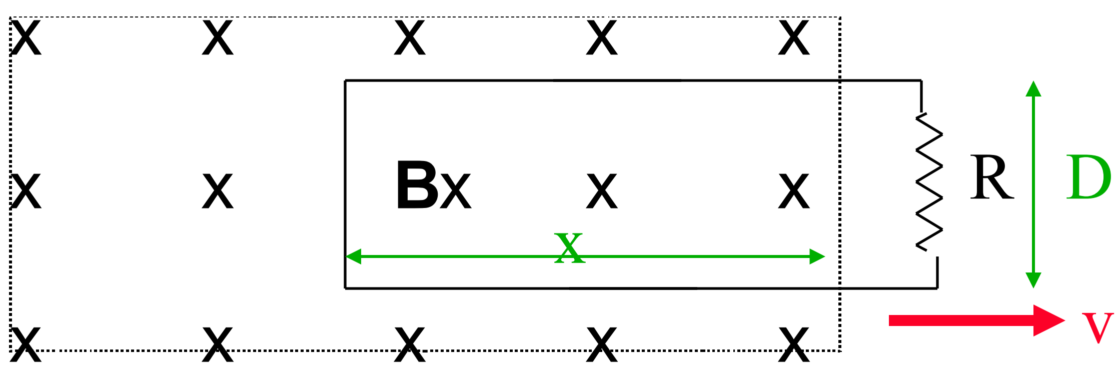
What is the direction of the induced current and field?

Motional EMF

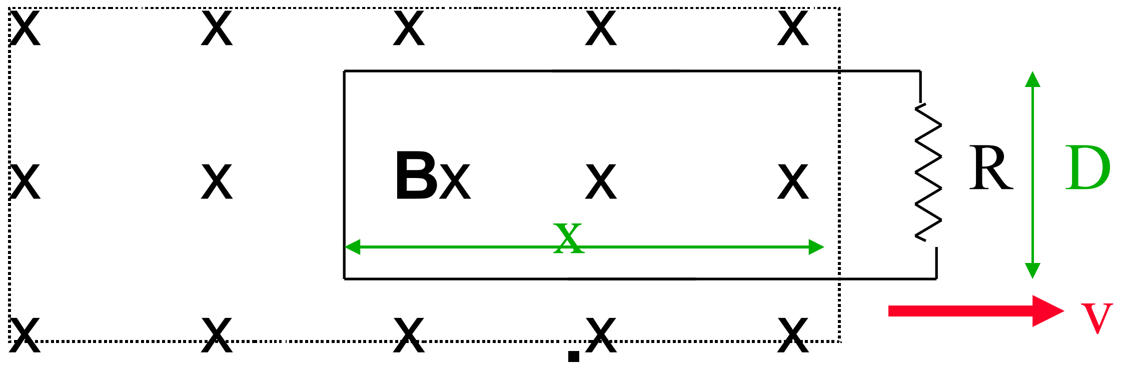
Up until now we have considered fixed loops. The flux through them changed because the magnetic field changed with time.

Now try moving the loop in a uniform and constant magnetic field. This changes the flux, too.

B points into screen



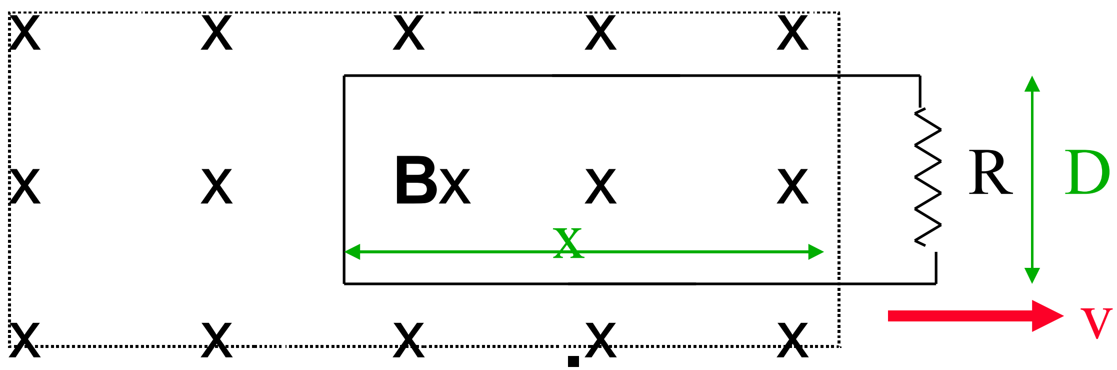
Motional EMF - Use Faraday's Law



The flux is $\Phi_B = \underline{B} \cdot \underline{A} = BDx$

This changes in time:

Motional EMF - Use Faraday's Law



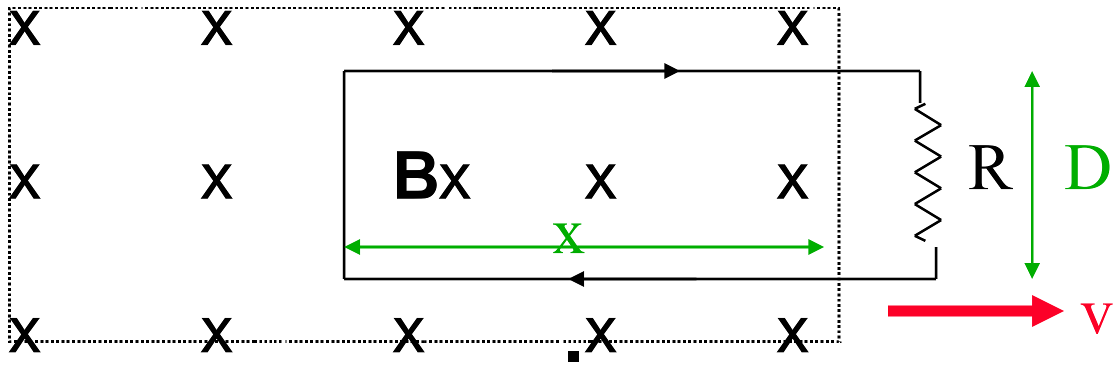
The flux is $\Phi_B = \underline{B} \cdot \underline{A} = BDx$

This changes in time:

$$d\Phi_B / dt = d(BDx)/dt = BDdx/dt = -BDv$$

Hence by Faraday's law there is an induced emf and current. What is the direction of the current?

Motional EMF - Use Faraday's Law



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This changes in time:

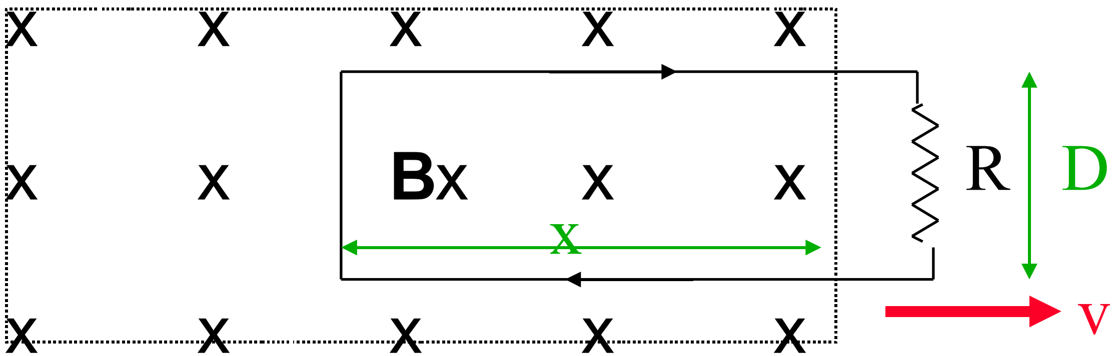
$$d\Phi_B / dt = d(BDx)/dt = BDdx/dt = -BDv$$

Hence by Faraday's law there is an induced emf and current. What is the direction of the current?

Lenz's law: there is less inward flux through the loop. Hence the induced current gives inward flux.

$\Rightarrow$ So the induced current is clockwise.

Motional EMF
Faraday's Law



Now Faraday's Law $d\Phi_B/dt = -\mathcal{E}$

gives the EMF $\Rightarrow \mathcal{E} = BDv$

In a circuit with a resistor, this gives

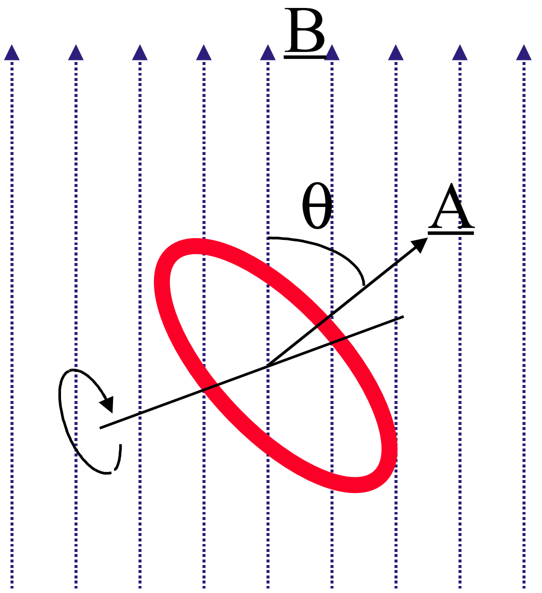
$\mathcal{E} = BDv = IR \Rightarrow I = BDv/R$

Thus moving a circuit in a magnetic field produces an emf exactly like a battery.
This is the principle of an electric generator.

Rotating Loop - The Electricity Generator

Consider a loop of area A in a region of space in which there is a uniform magnetic field B.
Rotate the loop with an angular frequency ω .

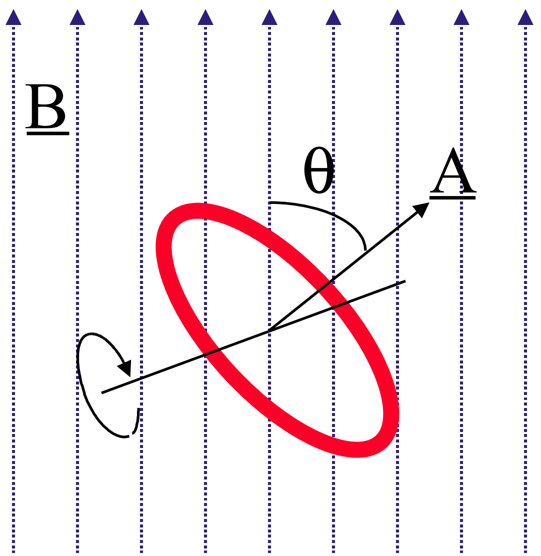
The flux changes because angle θ changes with time: $\theta = \omega t$.



Hence:

$$\begin{aligned} d\Phi_B/dt &= d(\underline{B} \,.\, \underline{A})/dt \\ &= d(BA\cos\theta)/dt \\ &= B\,A\,d(\cos(\omega t))/dt \\ &= -\,BA\omega\sin(\omega\,t) \end{aligned}$$

Rotating Loop - The Electricity Generator



$$d\Phi_B/dt = - BA\omega \sin(\omega t)$$

- Then by Faraday's Law this motion causes an emf

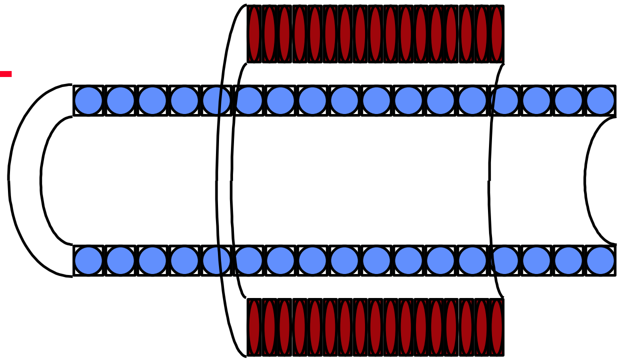
$$\mathcal{E} = - d\Phi_B /dt = BA\omega \sin(\omega t)$$

- This is an **<u>AC (alternating current) generator</u>**.

A New Source of EMF

- If we have a conducting loop in a magnetic field, we can create an EMF (like a battery) by changing the value of $\underline{B} \cdot \underline{A}$.
- This can be done by changing the area, by changing the magnetic field, or the angle between them.
- We can use this source of EMF in electrical circuits in the same way we used batteries.
- Remember we have to do work to move the loop or to change B, to generate the EMF (Nothing is for free!).

Example: a 120 turn coil (r= 1.8 cm, R = 5.3Ω) is placed outside a solenoid (r=1.6cm, n=220/cm, i=1.5A). The current in the solenoid is reduced to 0 in 0.16s. What current appears in the coil ?



Current induced in coil:

$$i_C = \frac{EMF}{R} = \left(\frac{N}{R}\right)\frac{d\phi_B}{dt}$$

$$\Phi_B = \vec{B}\cdot\vec{A} = (\mu_0 n i_s) A_s$$

Only field in coil is inside solenoid

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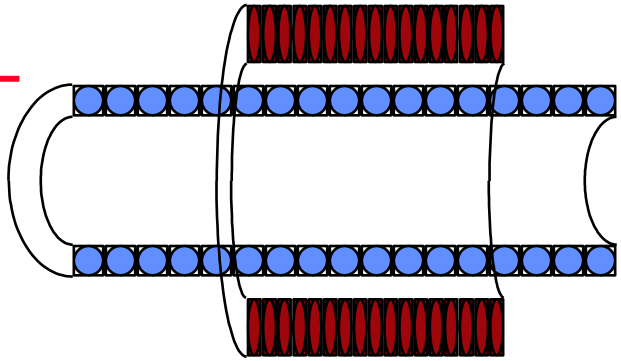

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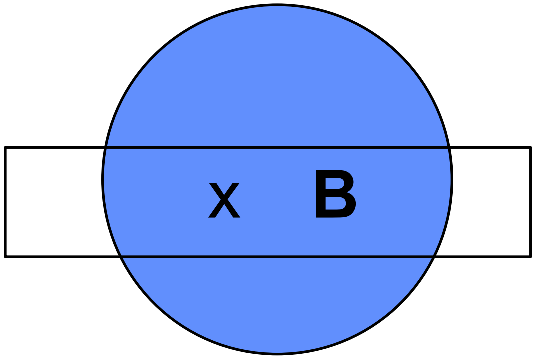
$$\Phi_B = \vec{B}\cdot\vec{A} = (\mu_0 n i_s)A_s$$

Only field in coil is inside solenoid

$$i_c = \left(\frac{N}{R}\right)\frac{d(\mu_0 n i_s A_s)}{dt} = \left(\frac{N}{R}\right)\mu_0 n A_s \frac{di_s}{dt}$$

$$= \left(\frac{N}{R}\right)\mu_0 n A \frac{i_0}{t} = 4.72\,mA$$

Induced Electric Fields



Consider a stationary conductor in a time-varying magnetic field. A current starts to flow.

So the electrons must feel a force $\underline{F}$.

It is not $\underline{F} = q\underline{v}\times\underline{B}$, because the wire is stationary.

Instead: we know that $\varepsilon = - d\Phi_B/dt$

This is equivalent to an induced electric field E, such that:

$$\underline{F} = q\underline{E} \quad \text{and} \quad \varepsilon = \oint \underline{E}.\underline{dl}$$

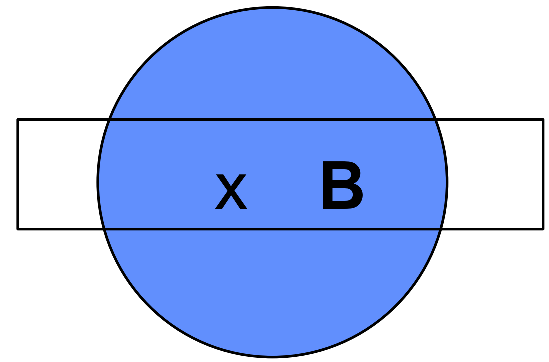
⇒ a time-varying magnetic field B causes an electric field E to appear in the conductor!

Induced Electric Fields

$$\mathcal{E} = \oint \underline{E} \,.\, \underline{dl}$$

and

$$\mathcal{E} = - d\Phi_B/dt$$



Then: $$\oint \underline{E} \,.\, \underline{dl} = - d\Phi_B/dt$$

Faraday's Law

The induced electric field E is NOT a conservative field

We can NOT write E = - dV/dl or $\underline{E} = -\underline{\nabla V}$

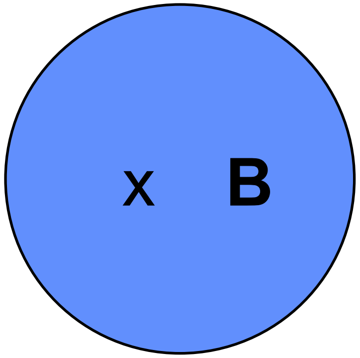
The electrostatic field E_e is conservative $\oint E_e . dl = 0$

Electrostatic Field	Induced Electric Field
$\underline{F} = q\,\underline{E}_e$	$\underline{F} = q\,\underline{E}$
$\Delta Vab = -\int \underline{E}_e . \underline{dl}$	$\int \underline{E} . \underline{dl} = - d\Phi_B/dt$
$\int \underline{E}_e . \underline{dl} = 0$ and $\underline{E}_e = \underline{\nabla V}$	$\int \underline{E} . \underline{dl} \neq 0$
Conservative	Nonconservative
Work or energy difference does NOT depend on path	**Work or energy difference DOES depend on path**
Caused by stationary charges or emf sources	**Caused by changing magnetic fields**

Induced Electric Fields

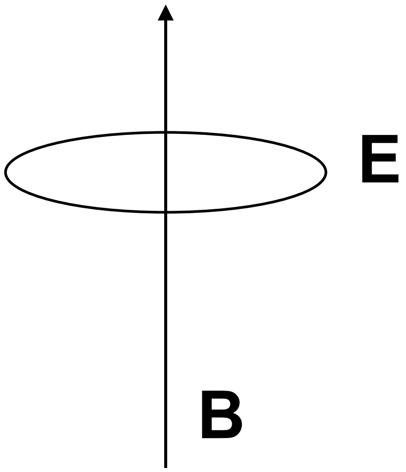
$$\oint \underline{E} \cdot \underline{dl} = - d\Phi_B/dt$$

Faraday's Law



Now suppose there is no conductor:
Is there still an electric field?

YES!, the field does not depend on the presence of the conductor.



For a magnetic field with axial or cylindrical symmetry, the field lines of E are circles.