Sources of Magnetic Fields

Chapter 30

Biot-Savart Law
Lines of Magnetic Field
Ampere's Law
Solenoids and Toroids

Sources of Magnetic Fields

- We saw that magnetic fields, from permanent magnets, exert forces on moving charges.
- It turns out that something reciprocal happens: moving charges give rise to magnetic fields (which can then exert a force on other moving charges).
- We'll describe the magnetic field created by currents in wires, the easiest case; but the magnetism of permanent magnets also comes from moving charges (the electrons in the atoms).

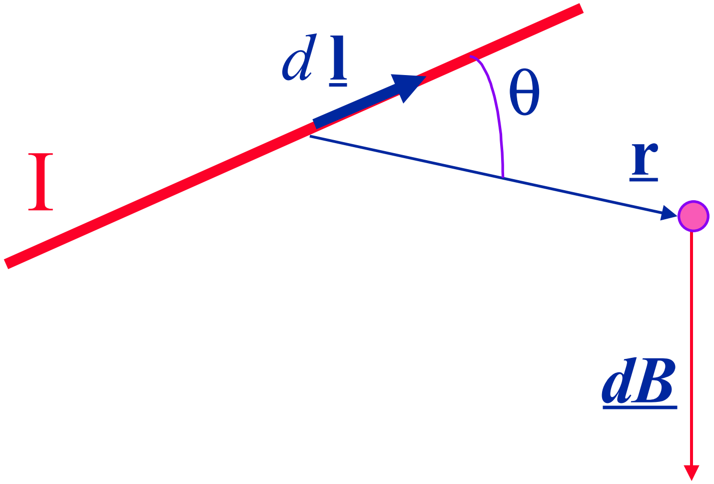
Magnetic Interaction

In this chapter we learn that:

- A current generates a magnetic field.
- A magnetic field exerts a force on a current.
- Two contiguous conductors, carrying currents, will exert forces on each other.

Biot-Savart Law

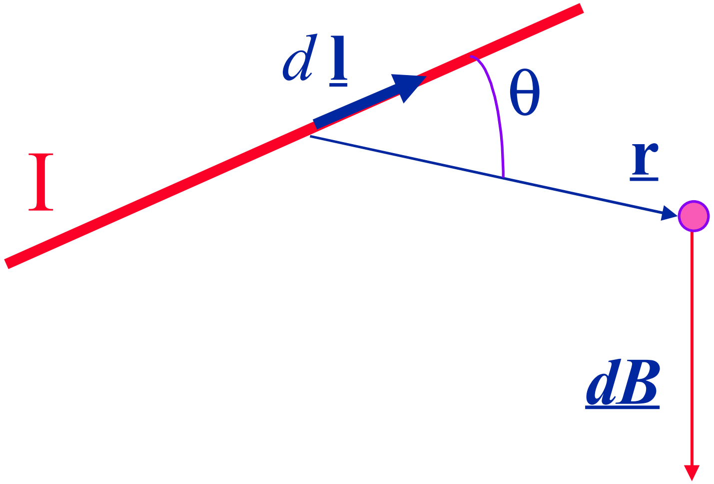
- The mathematical description of the magnetic field B due to a current-carrying wire is called the **Biot-Savart** law. It gives **B** at a selected position.
- A current I is moving all through the wire. We need to add up the bits of magnetic field *d***B** arising from each infinitesimal length d**l**.



$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l} \times \hat{r}}{r^2}$$

Add up all the bits!

Biot-Savart Law



$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I d\vec{l} \times \hat{r}}{r^2}$$

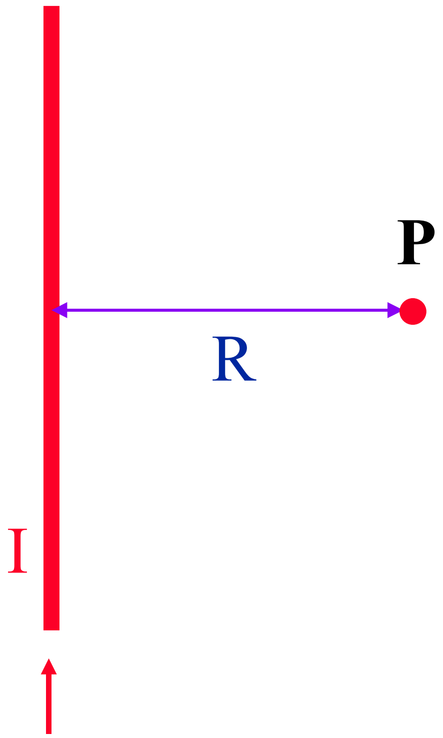
The constant $\mu_0 = 4\pi \times 10^{-7}$ T m/A is called the permeability of free space.

It turns out that μ_0 and ε_o are related in a simple way: $(\varepsilon_0\mu_0)^{-1/2} = 3\text{x}10^{8}$ m/s = c, the speed of light.

Why? Light is a wave of electric and magnetic fields.

Example: Magnetic field from a long wire

Consider a long straight wire carrying a current I. We want to find the magnetic field **B** at a point P, a distance R from the wire.



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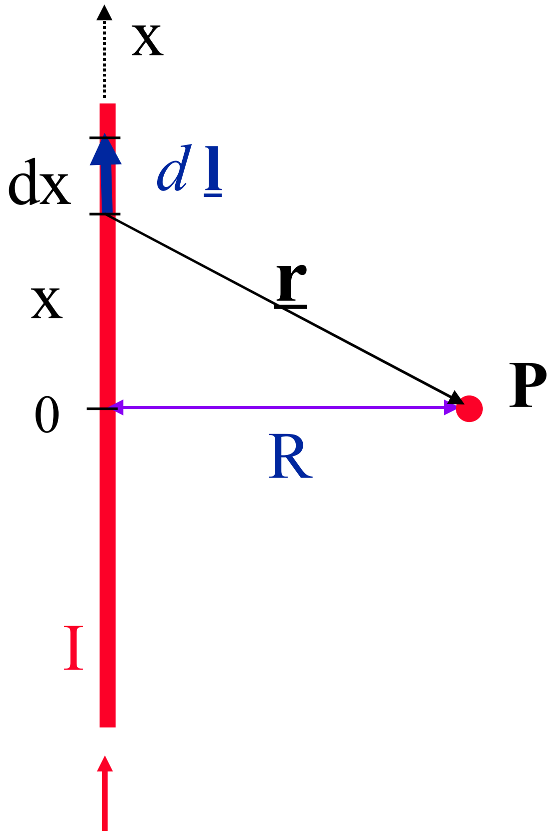
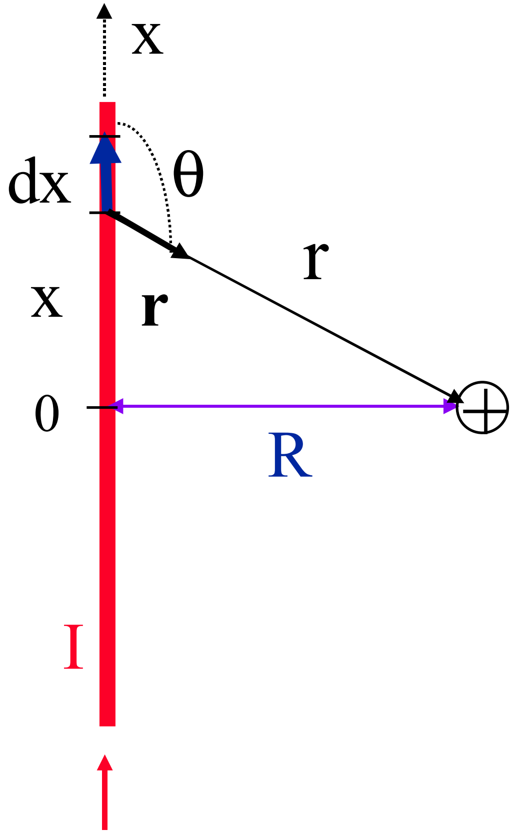


Break the wire into bits $d\underline{\mathbf{l}}$. To do that, choose coordinates: let the wire be along the x axis, and consider the little bit dx at a a position x.

The vector $\underline{\mathbf{r}} = r\,\hat{r}$ is from this bit to the point P.

Example: Magnetic field from a long wire



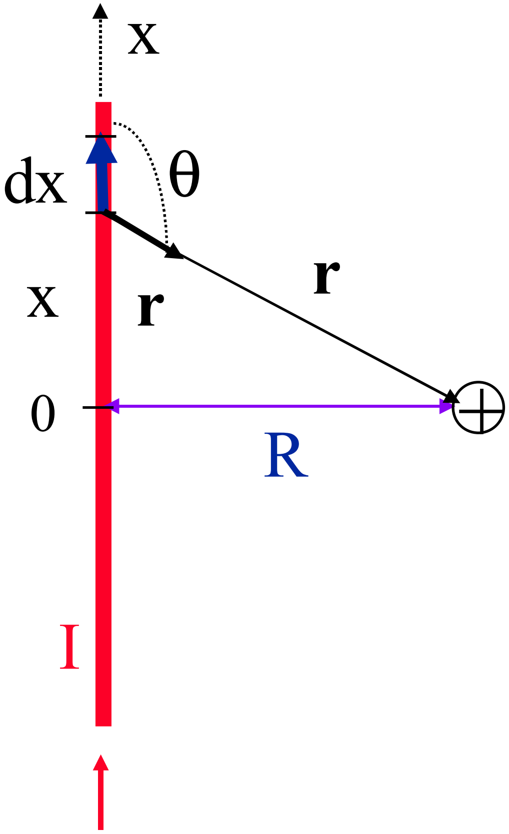
$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{\mathbf{l}} \, x \, \hat{\mathbf{r}}}{r^2}$$

Direction of dB (or B): into page

$$dB = \frac{\mu_0 I}{4\pi} \frac{dx \sin\theta}{r^2}$$

$$\therefore \quad B = \int dB = \frac{\mu_0 I}{4\pi} \int_{x=-\infty}^{x=+\infty} \frac{\sin\theta \, dx}{r^2}$$

Example: Magnetic field from a long wire



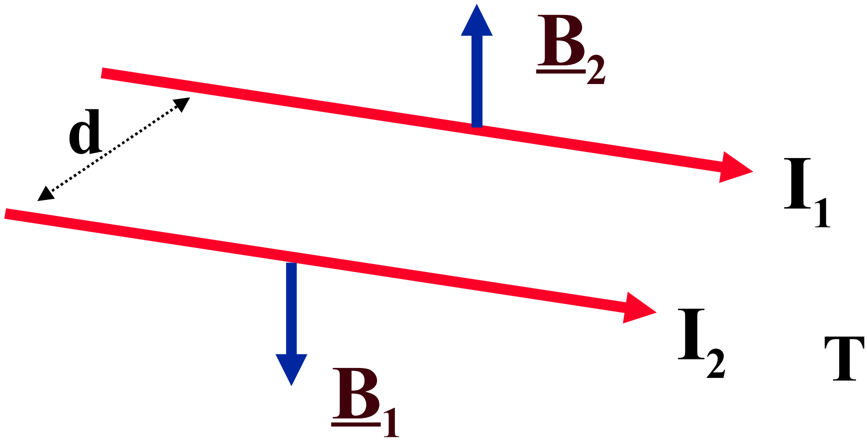
$$\therefore \; B = \int dB = \frac{\mu_0 I}{4\pi} \int_{x=-\infty}^{x=+\infty} \frac{\sin\theta \, dx}{r^2}$$

$$r = \sqrt{x^2 + R^2}, \;\; \sin\theta = \frac{R}{r}$$

$$B = \frac{\mu_0 I}{4\pi} \int_{x=-\infty}^{x=+\infty} \frac{R\,dx}{(x^2 + R^2)^{3/2}}$$

$$= \frac{\mu_0 I}{4\pi R} \frac{x dx}{(x^2 + R^2)^{1/2}} \Bigg|_{x=-\infty}^{x=+\infty} = \frac{\mu_0 I}{2\pi R}$$

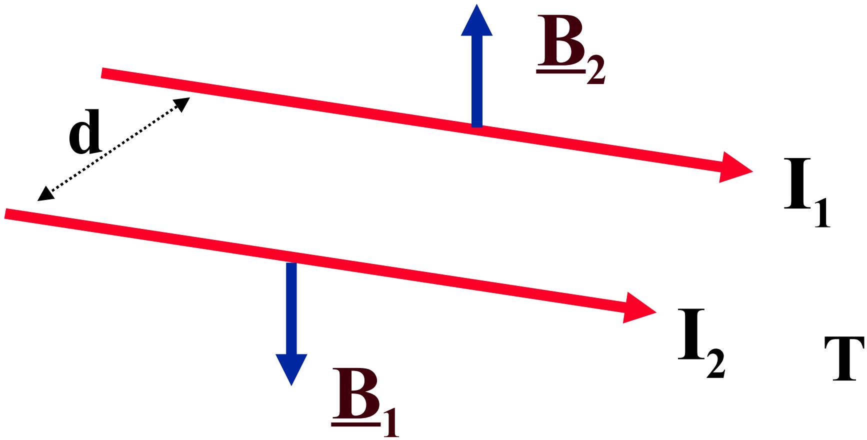
Force between two current-carrying wires



Current 1 produces a magnetic field $B_1 = \mu_0 I / (2\pi\, d)$ at the position of wire 2.

This produces a force on current 2:

Force between two current-carrying wires



Current 1 produces a magnetic field $B_1 = \mu_0 I / (2\pi\, d)$ at the position of wire 2.

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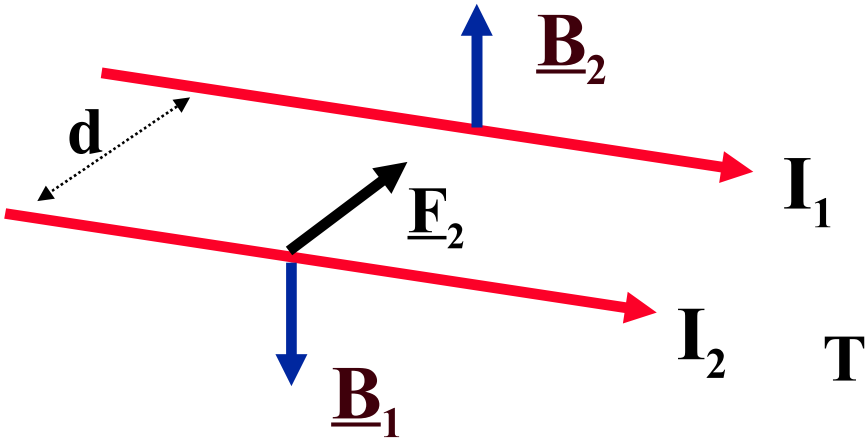
$$\underline{F}_2 = I_2 \underline{L} \times \underline{B}_1$$

Force between two current-carrying wires



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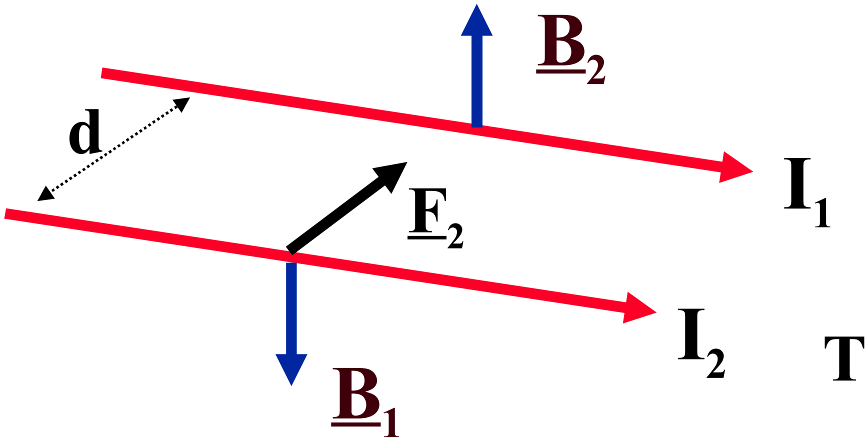

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This produces a force on current 2:

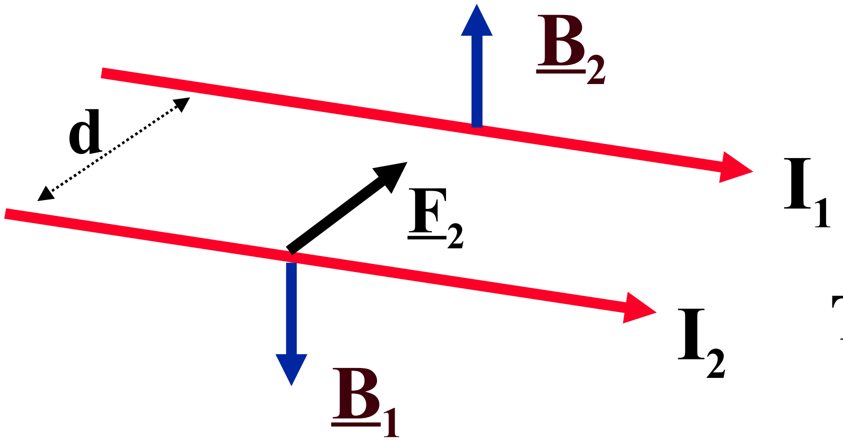
$$\underline{F}_2 = I_2 \underline{L} \times \underline{B}_1$$

This gives the force on a length L of wire 2 to be:

$$F_2 = I_2 L B_1 = \frac{\mu_0 I_1 I_2 L}{2\pi\, d}$$

Direction: towards 1, if the currents are in the same direction.

Force between two current-carrying wires



Current I_1 produces a magnetic field $B_1 = \mu_0 I/(2\pi\, d)$ at the position of the current I_2.

This produces a force on current I_2:

$$\underline{F}_2 = I_2 \underline{L} \times \underline{B}_1$$

Thus, the force on a length L of the conductor 2 is given by:

$$F_2 = I_2 L B_1 = \frac{\mu_0 I_1 I_2 L}{2\pi\ d}$$

[Direction: towards I_1]

The magnetic force between two parallel wires carrying currents in the same direction is attractive .

What is the force on wire 1?. What happens if one current is reversed?

Magnetic field from a circular current loop

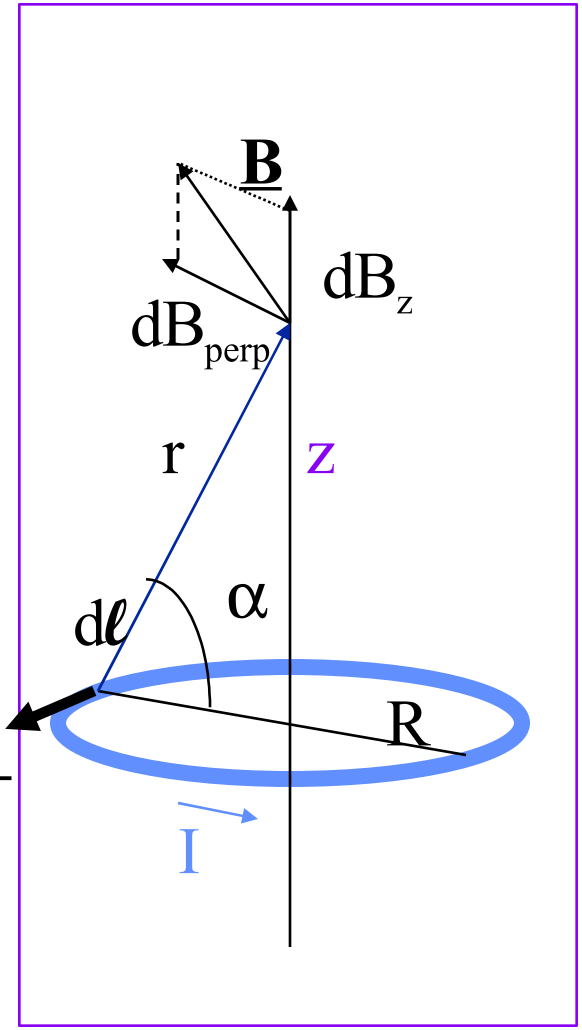
$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{\mathbf{l}} \times \hat{\mathbf{r}}}{r^2}$$

$$dB = \frac{\mu_0}{4\pi}\frac{Idl}{r^2}$$

Only z component is nonzero.

$$dB_z = dB\cos\alpha = \frac{\mu_0}{4\pi}\frac{Idl\cos\alpha}{r^2}$$

$$r = \sqrt{R^2 + z^2}, \cos\alpha = R \Big/ \sqrt{R^2 + z^2}$$

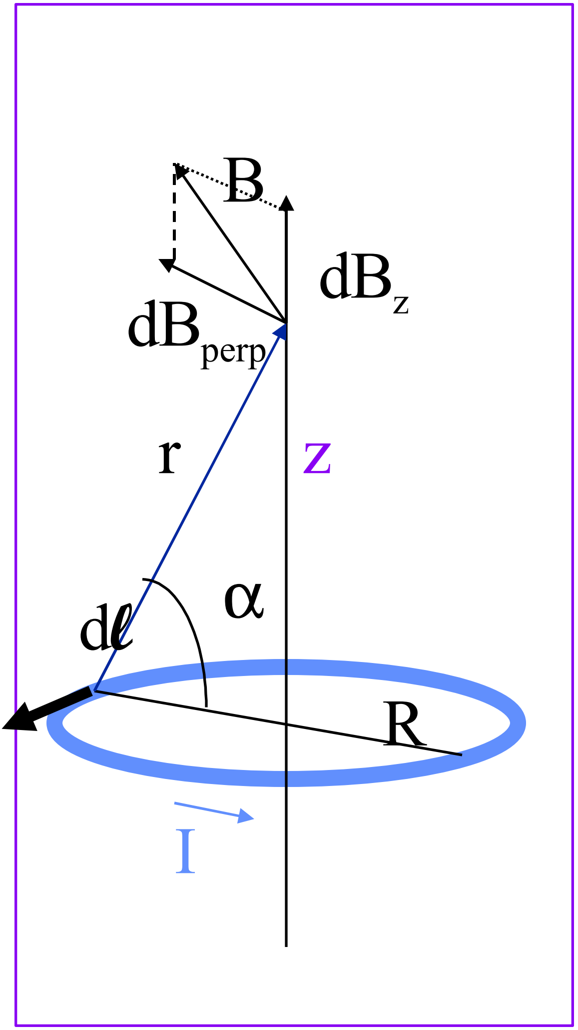


Magnetic field from a circular current loop

$$B = \int dB_z = \int \frac{\mu_0}{4\pi} \frac{IR}{(R^2 + z^2)^{3/2}} \, dl$$

$$B = \frac{\mu_0}{4\pi} \frac{IR}{(R^2 + z^2)^{3/2}} \int dl =$$

$$B = \frac{\mu_0}{4\pi} \frac{IR}{(R^2 + z^2)^{3/2}} \, 2\pi R = \frac{\mu_0}{2} \frac{IR^2}{(R^2 + z^2)^{3/2}}$$



At the center of the loop $B = \frac{\mu_0 I}{2R}$

At distance z on axis from the loop, z>>R $B = \frac{\mu_0 I R^2}{2z^3}$

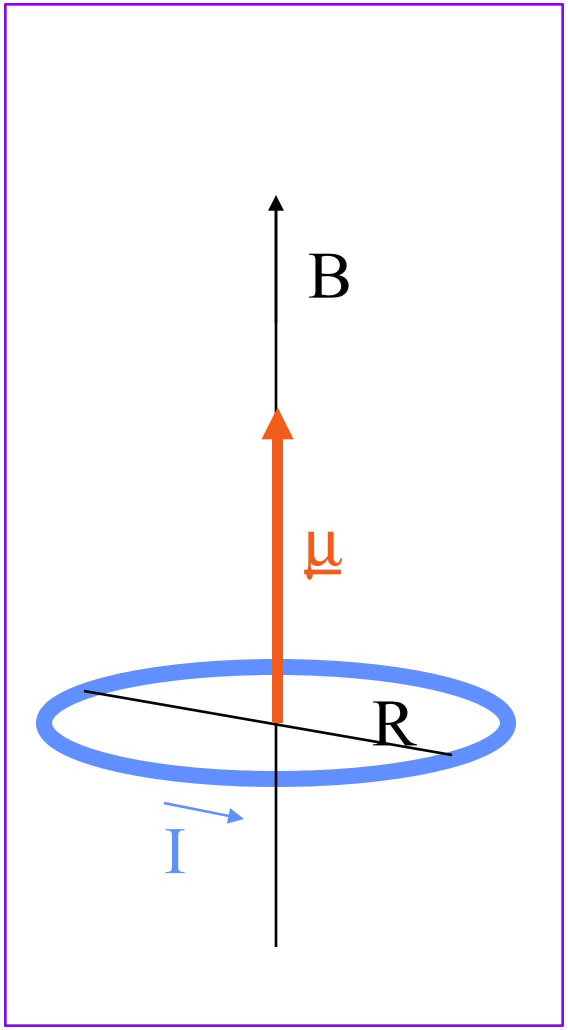
Magnetic field in terms of dipole moment

Far away on the axis,

$$B = \frac{\mu_0 I R^2}{2z^3}$$

The magnetic dipole moment of the loop is defined as μ = IA = IπR².

The direction is given by the right hand rule: with fingers closed in the direction of the current flow, the thumb points along **μ**.



Magnetic field in terms of dipole moment

In terms of μ, the magnetic field on axis (far from the loop) is therefore

$$B = \frac{\mu_0 \mu}{2\pi z^3}$$

This also works for a loop with N turns. Far from the loop the same expression is true with the dipole moment given by $\mu = NIA = I\pi NR^2$

Dipole Equations

Electric Dipole

$\underline{\tau} = \underline{\mathbf{p}} \;\; x \;\; \underline{\mathbf{E}}$

$U \;=\; - \underline{\mathbf{p}} \,.\, \underline{\mathbf{E}}$

$E_{ax} \;=\; (2\pi\varepsilon_0)^{-1} \;\; p/z^3$

$E_{bis} \;=\; (4\pi\varepsilon_0)^{-1} \;\; p/x^3$

Magnetic Dipole

$\underline{\tau} = \underline{\boldsymbol{\mu}} \;\; x \;\; \underline{\mathbf{B}}$

$U \;=\; - \underline{\boldsymbol{\mu}} \,.\, \underline{\mathbf{B}}$

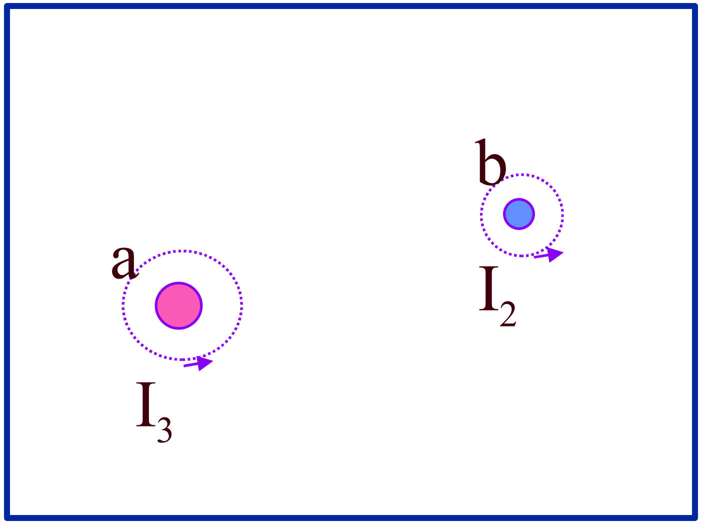
$B_{ax} \;=\; (\mu_0/2\pi) \;\; \mu/z^3$

$B_{bis} \;=\; (\mu_0/4\pi) \;\; \mu/x^3$

Ampere's Law

Draw an "Amperian loop" around the sources of current.

blue - into figure (-)
red - out of figure (+)



The line integral of the tangential component of **B** around this loop is equal to $\mu_o I$:

$$\oint \boldsymbol{B} \bullet d\vec{l} = \mu_0 I$$

Ampere's law is to the Biot-Savart law exactly what Gauss's law is to Coulomb's law.

Ampere’s Law - a line integral

$$\oint_a \vec{B}\cdot d\vec{l} =$$

$$\oint_b \vec{B}\cdot d\vec{l} =$$

$$\oint_c \vec{B}\cdot d\vec{l} =$$

$$\oint_d \vec{B}\cdot d\vec{l} =$$

blue - into figure (-)
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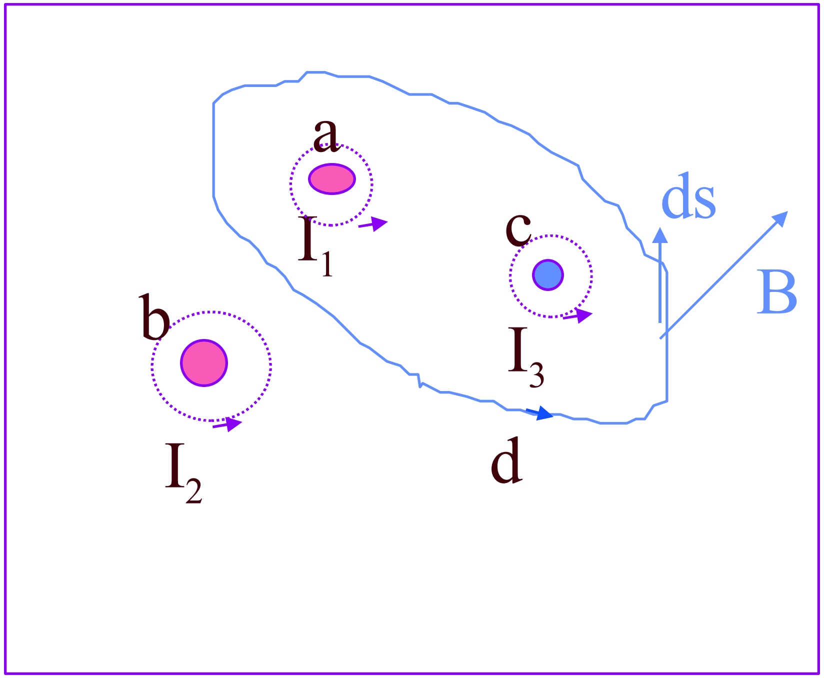


Ampere's Law - a line integral

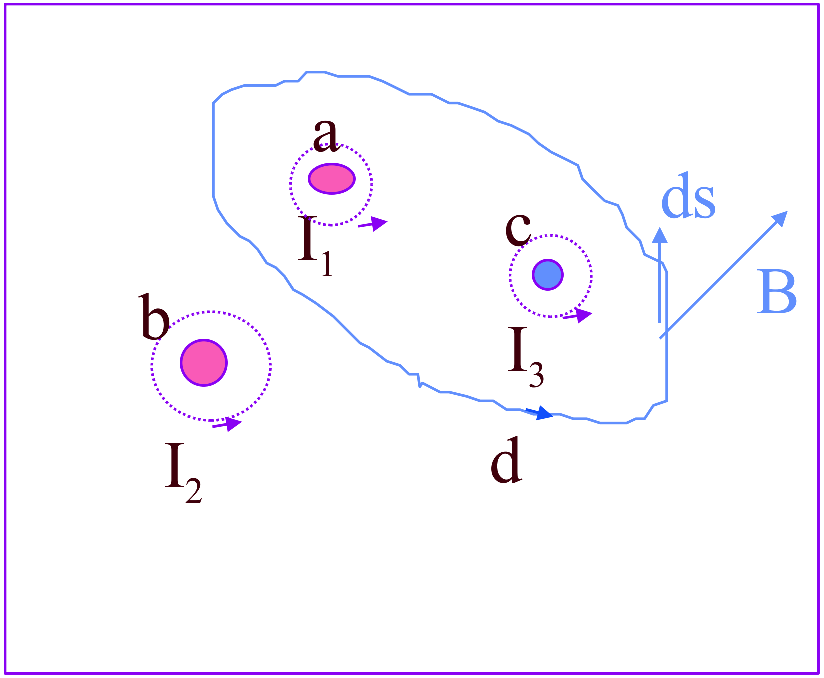
$$\oint_a \vec{B}\cdot d\vec{l} = \mu_0 I_1$$

$$\oint_b \vec{B}\cdot d\vec{l} =$$

$$\oint_c \vec{B}\cdot d\vec{l} =$$

$$\oint_d \vec{B}\cdot d\vec{l} =$$

blue - into figure (**-** *)*
red - out of figure (+)



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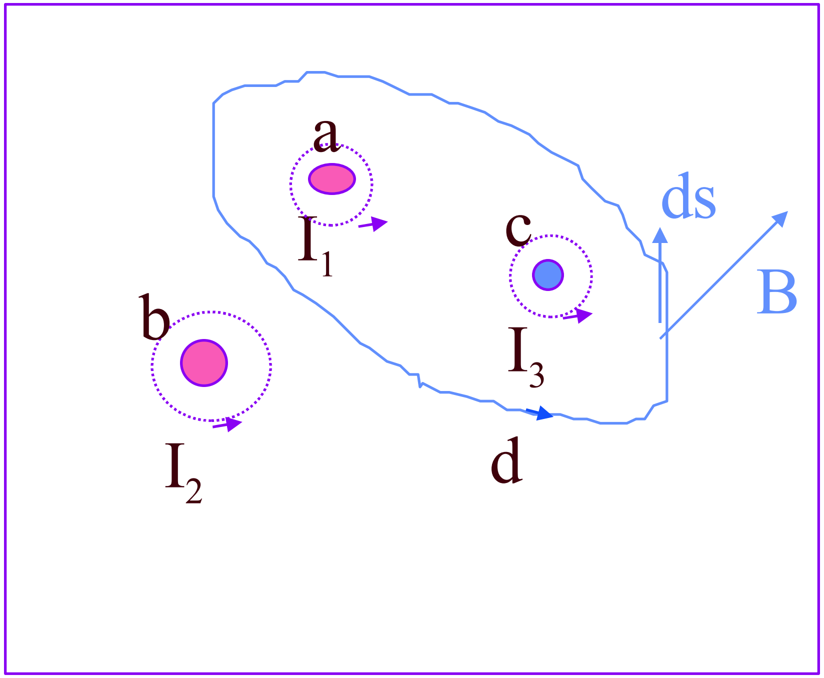

Ampere's Law - a line integral

$$\oint_a \vec{B}\cdot d\vec{l} = \mu_0 I_1$$

$$\oint_b \vec{B}\cdot d\vec{l} = \mu_0 I_2$$

$$\oint_c \vec{B}\cdot d\vec{l} = \mu_0 \left(-I_3\right)$$

$$\oint_d \vec{B}\cdot d\vec{l} =$$

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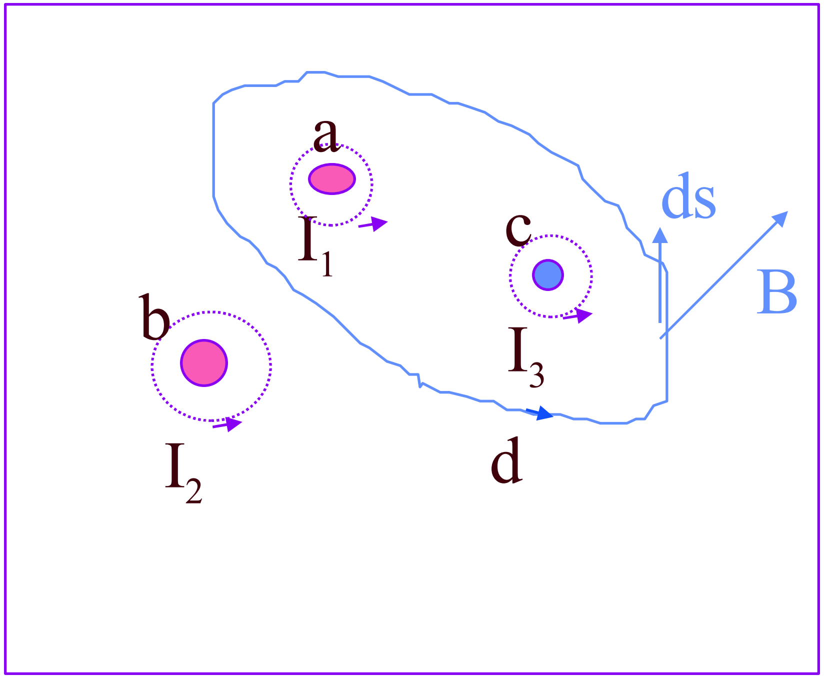

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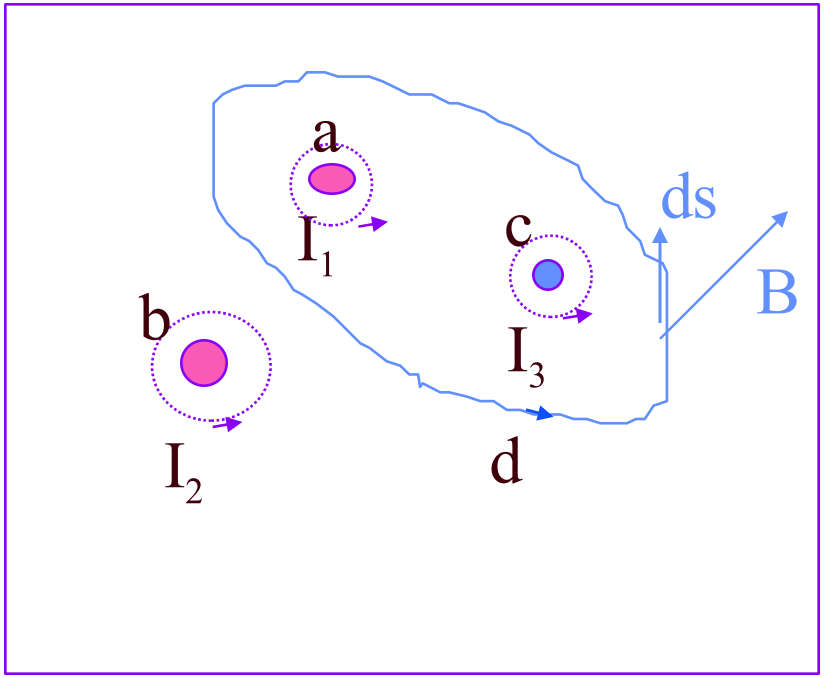
$$\oint_b \vec{B}\cdot d\vec{l} = \mu_0 I_2$$

$$\oint_c \vec{B}\cdot d\vec{l} = \mu_0 (-I_3)$$

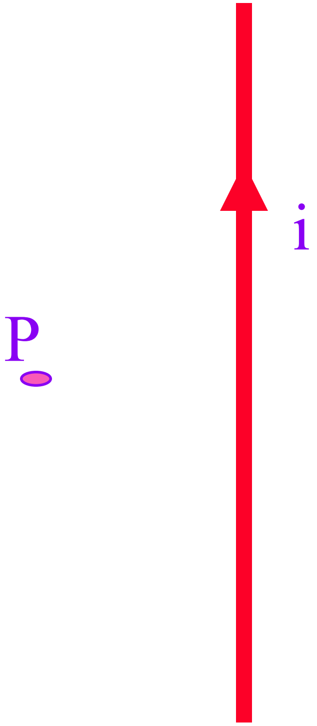
$$\oint_d \vec{B}\cdot d\vec{l} = \mu_0 (I_1 - I_3)$$

blue - into figure (- *)*
red - out of figure (+)



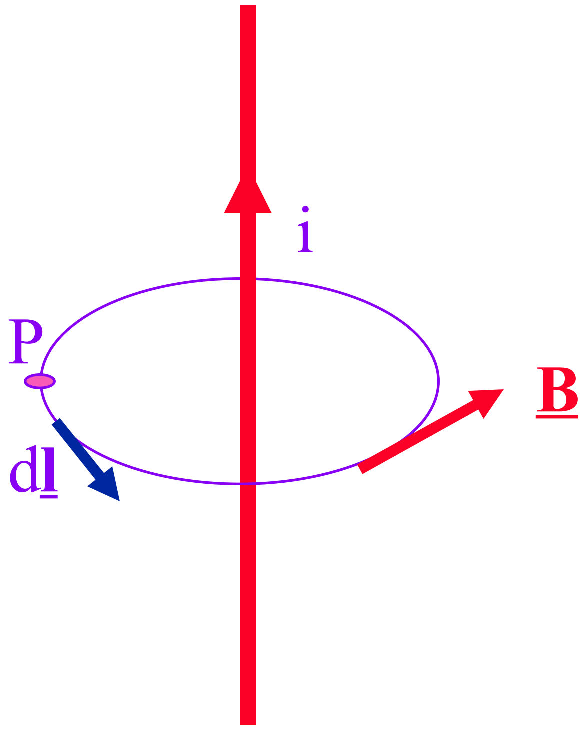
Ampere’s Law on a Wire

What is magnetic field at point P ?



Ampere's Law on a Wire

What is magnetic field at point P ? Draw Amperian loop through P around current source and integrate **B** . dl around loop



HINT: TAKE ADVANTAGE OF SYMMETRY!!!!

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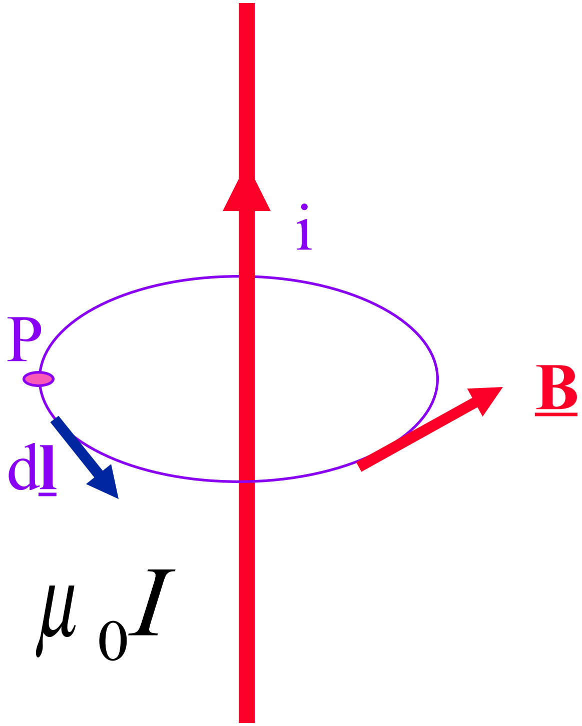


Then

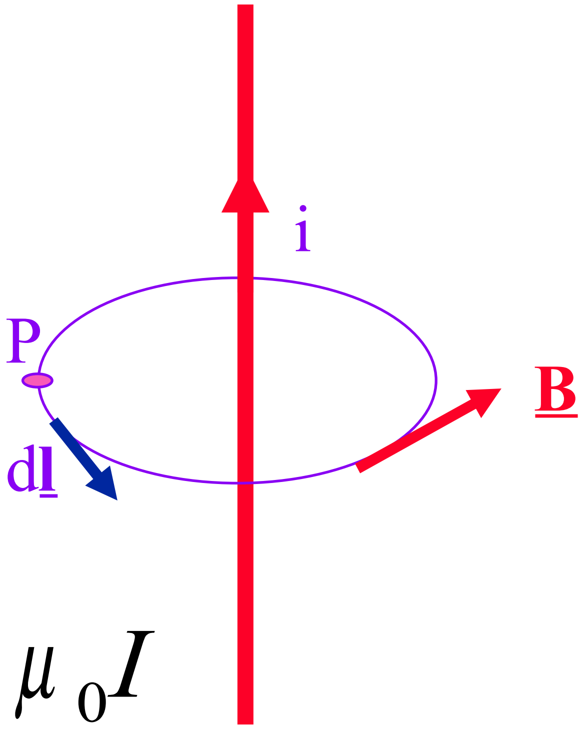
$$\oint \vec{B} \bullet d\vec{l} = \oint B\,dl = B(2\pi\, r) = \mu_0 I$$

HINT: TAKE ADVANTAGE OF SYMMETRY!!!!

Ampere’s Law for a Wire

What is the magnetic field at point P?.

Draw an Amperian loop through P, around the current source, and integrate **B** . dl around the loop. Then:

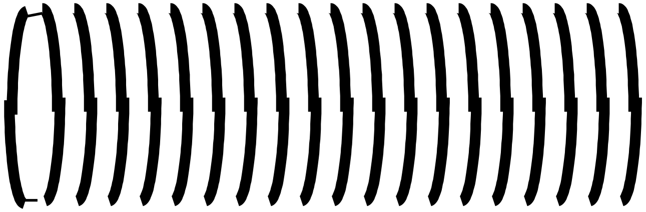


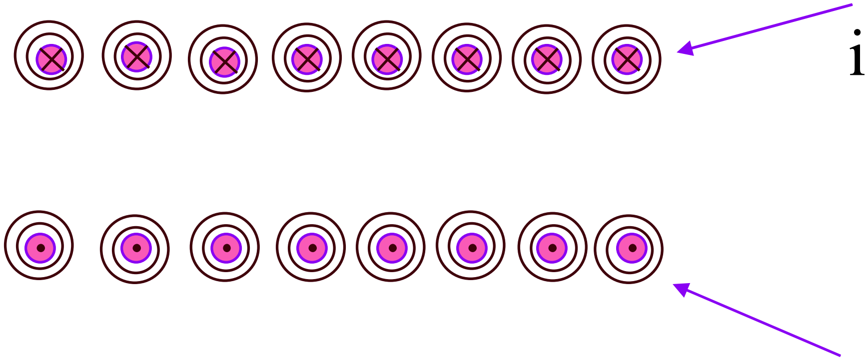
$$\oint \boldsymbol{B} \bullet d\vec{l} = \oint B\,dl = B(2\pi r) = \mu_0 I$$

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

A Solenoid

.. is a closely wound coil having n turns per unit length.





current flows
into plane

current flows
out of plane

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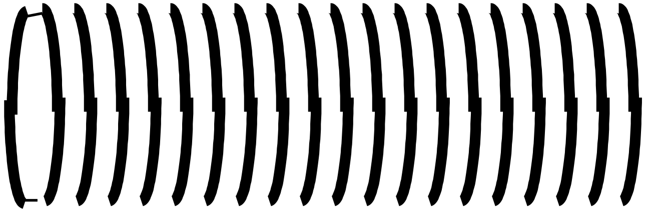

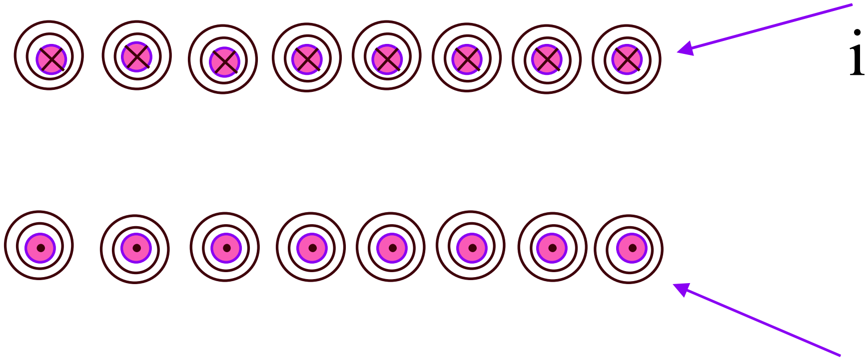

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current flows out of plane

What direction is the magnetic field?

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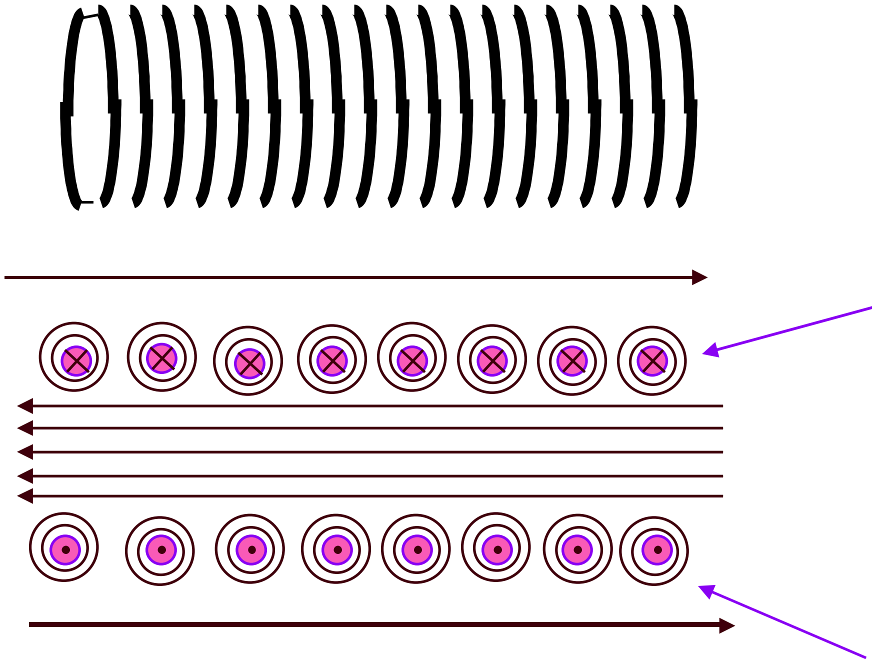


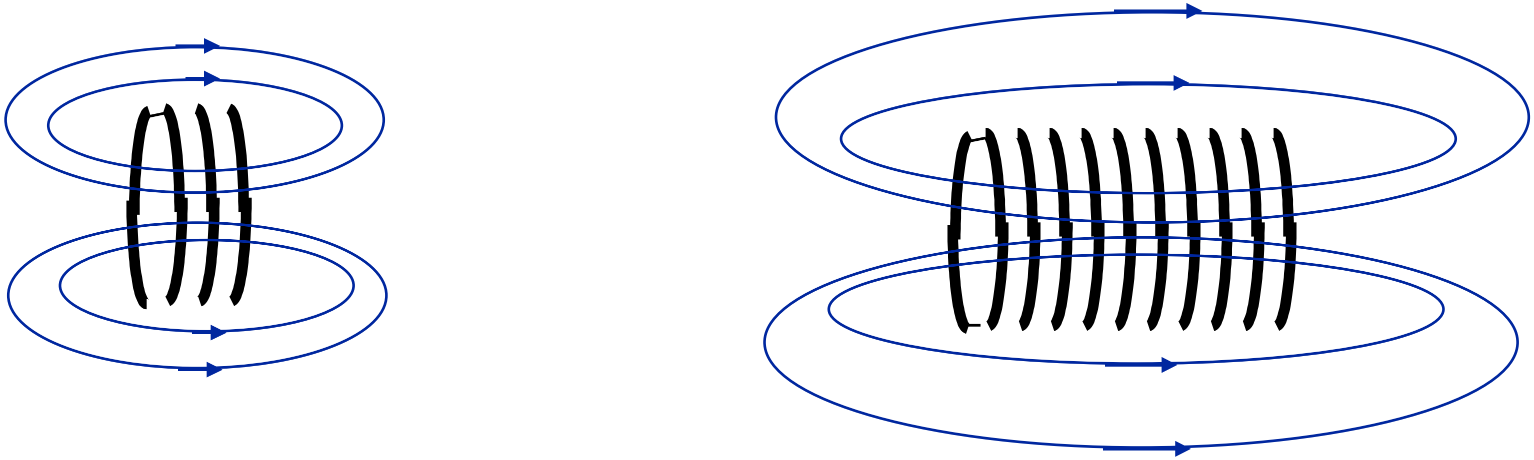
current flows into plane

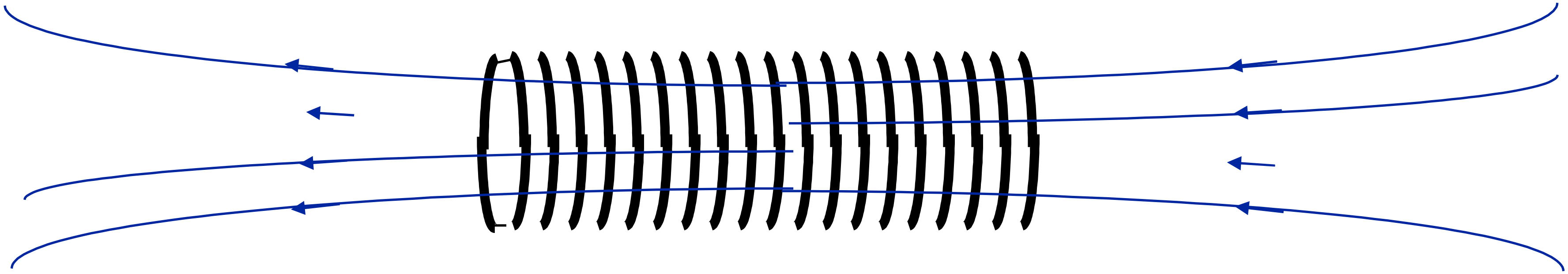
current flows out of plane

What direction is the magnetic field?

A Solenoid

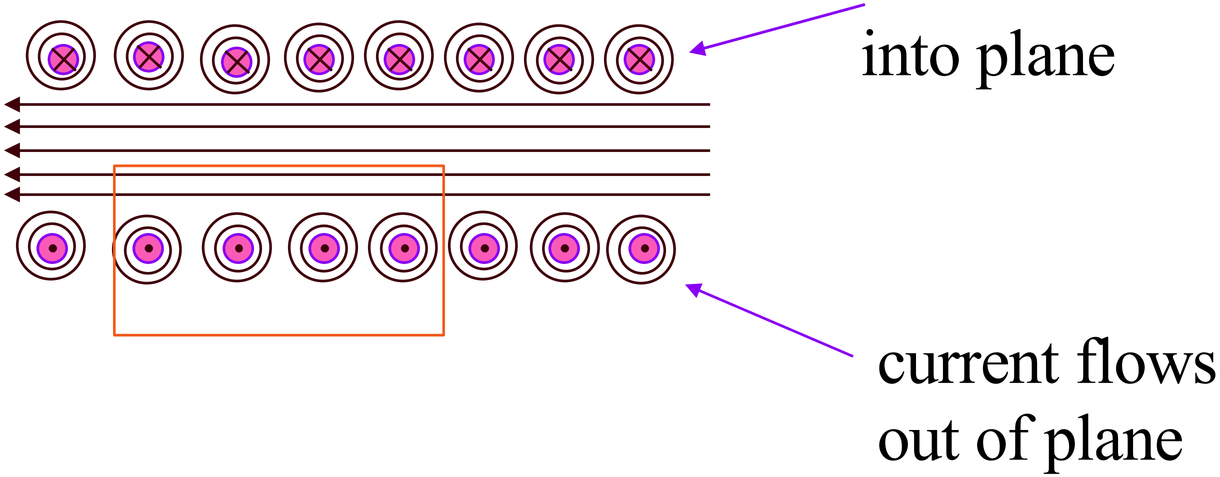
Consider longer and longer solenoids.





Fields get weaker and weaker outside.

Apply Ampere’s Law to the loop shown.
Is there a net enclosed current?
In what direction does the field point?
What is the magnetic field inside the solenoid?



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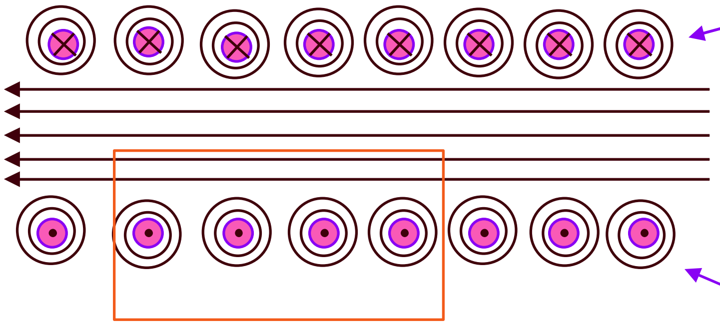

current flows into plane

current flows out of plane

$$B(L) = \mu_0 (nLI) \quad \Rightarrow \quad B = n\mu_0 I$$

Solenoids and Toroids

- Solenoid $B = \mu_o In$ n = # of turns/m of length of the solenoid

  This is valid inside, not too near the ends.

- A toroid is a solenoid bent in a circle.

  A similar calculation gives $B = \mu_I IN/2\pi r$,

  where in this case N is the total number of turns.

Gauss’s Law for Magnetism

For electric charges
Gauss’s Law is:

$$\oint \underline{E} \bullet \underline{dA} = {}^{q}\!/_{\varepsilon_0}$$

because there are single electric charges. On the other hand, we have never detected a single magnetic charge, only dipoles. Since there are no magnetic monopoles there is no place for magnetic field lines to begin or end.

Thus, Gauss’s Law for
magnetic charges must be:

$$\boxed{\oint \underline{B} \bullet \underline{dA} = 0}$$

Laws of Electromagnetism

We have now 2.5 of Maxwell's 4 fundamental laws of electromagnetism. They are:

Gauss's law for electric charges
Gauss's law for magnetic charges
Ampere's law (it is still incomplete as it only applies to steady currents in its present form. Therefore, the 0.5 of a law.)

Magnetic Materials

The phenomenon of magnetism is due mainly to the orbital motion of electrons inside materials, as well as to the intrinsic magnetic moment of electrons (spin).

There are three types of magnetic behavior in bulk matter:

- Ferromagnetism
- Paramagnetism
- Diamagnetism

Magnetic Materials

Because of the configuration of electron orbits in atoms, *and* due to the intrinsic magnetic properties of electrons and protons (called “spin”), materials can enhance or diminish applied magnetic fields:

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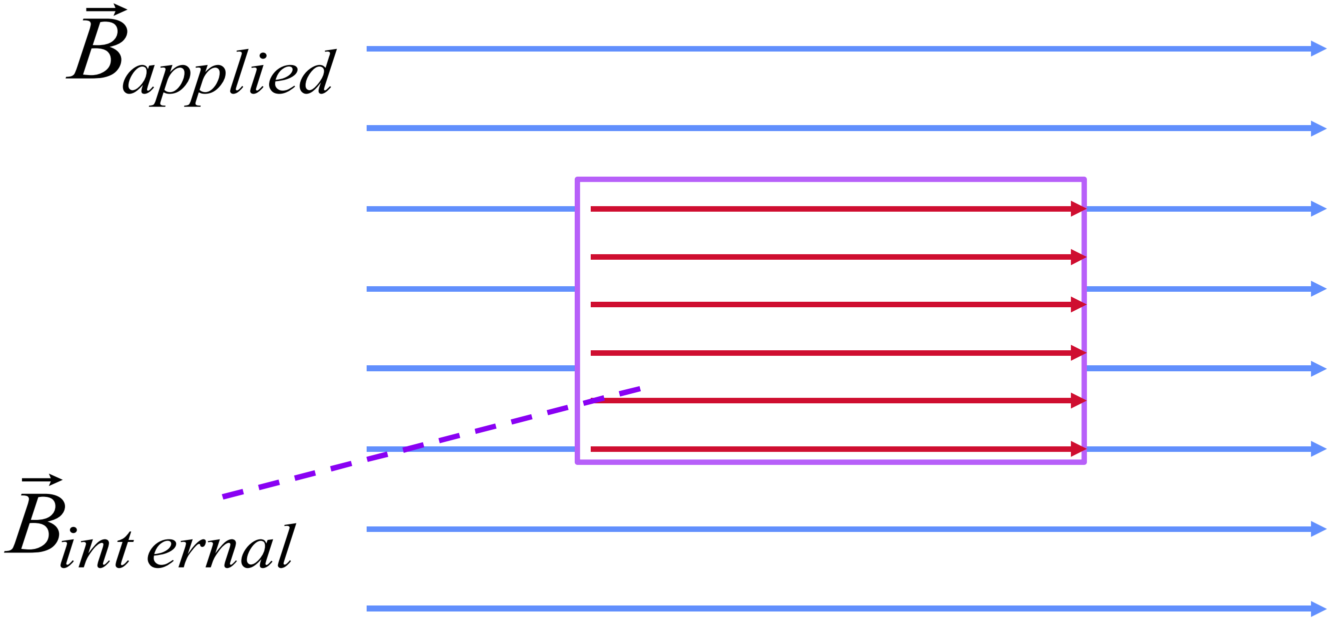

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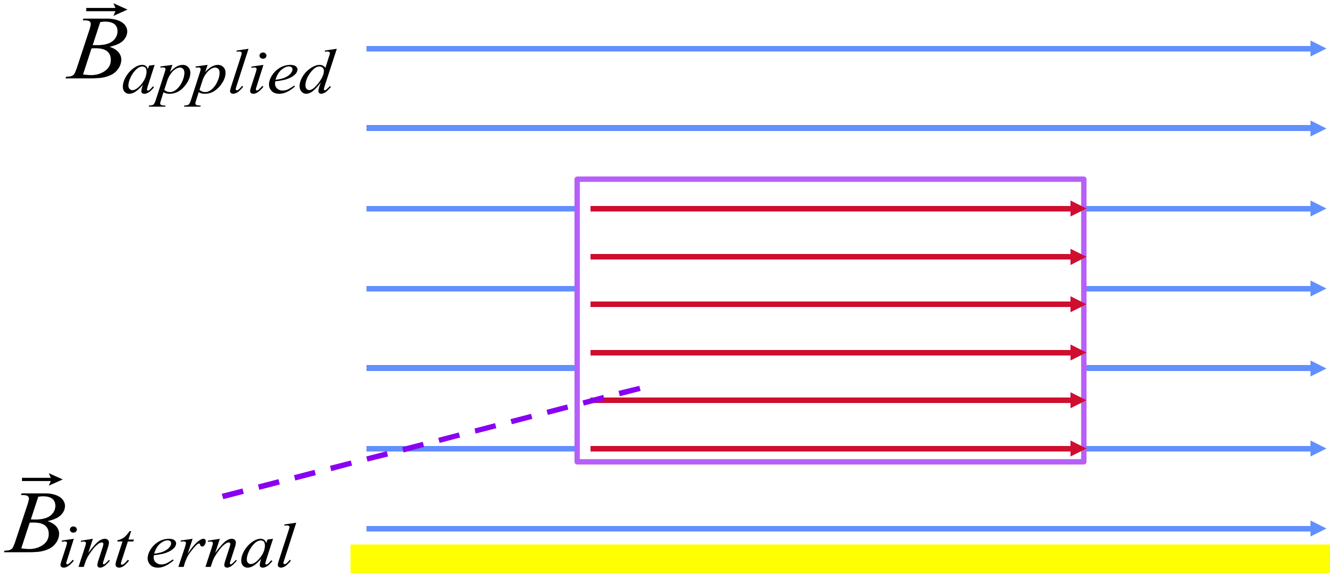

$$\vec{B}_{int} = \kappa_M \vec{B}_{app}$$

Magnetic Materials

$$\vec{B}_{int} = \kappa_M \vec{B}_{app}$$

κ_M is the “relative permeability”:

(It is the magnetic equivalent of κ_E*)*

Usually κ_M is very close to 1.

- if $\kappa_M > 1$, material is “paramagnetic” - e.g. O_2
- if $\kappa_M < 1$, material is “diamagnetic” - e.g. Cu

Because κ_M is close to 1, we define the “magnetic susceptibility”, $\chi_M = \kappa_M - 1$

Magnetic Materials

$$\vec{B}_{\text{int}} = \kappa_M \vec{B}_{app} = (1 + \chi_M)\vec{B}_{app}$$

Hence:

For *paramagnetic* materials χ_M is *positive*

- so $B_{\text{int}} > B_{\text{app}}$

For *diamagnetic* materials χ_M is *positive*

- so $B_{\text{int}} < B_{\text{app}}$

Typically, $\chi_M \sim +10^{-5}$ for paramagnetics,

$\chi_M \sim -10^{-6}$ for diamagnetics.

(For both κ is very close to 1)

Magnetic Materials

Ferromagnetic Materials:

These are the stuff permanent magnets are made of.

These materials can have huge susceptibilities:

χ_M as big as $+10^4$

Magnetic Materials

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