Magnetic Fields
Chapter 29

Permanent Magnets & Magnetic Field Lines

The Magnetic Force on Charges

Magnetism

- Our most familiar experience of magnetism is through **permanent magnets.**
- These are made of materials which exhibit a property we call **"ferromagnetism"** - i.e., they can be magnetized.
- Depending on how we position two magnets, they will attract or repel, ***i.e.* they exert forces on each other.**
- Thus, a magnet must have an associated field:

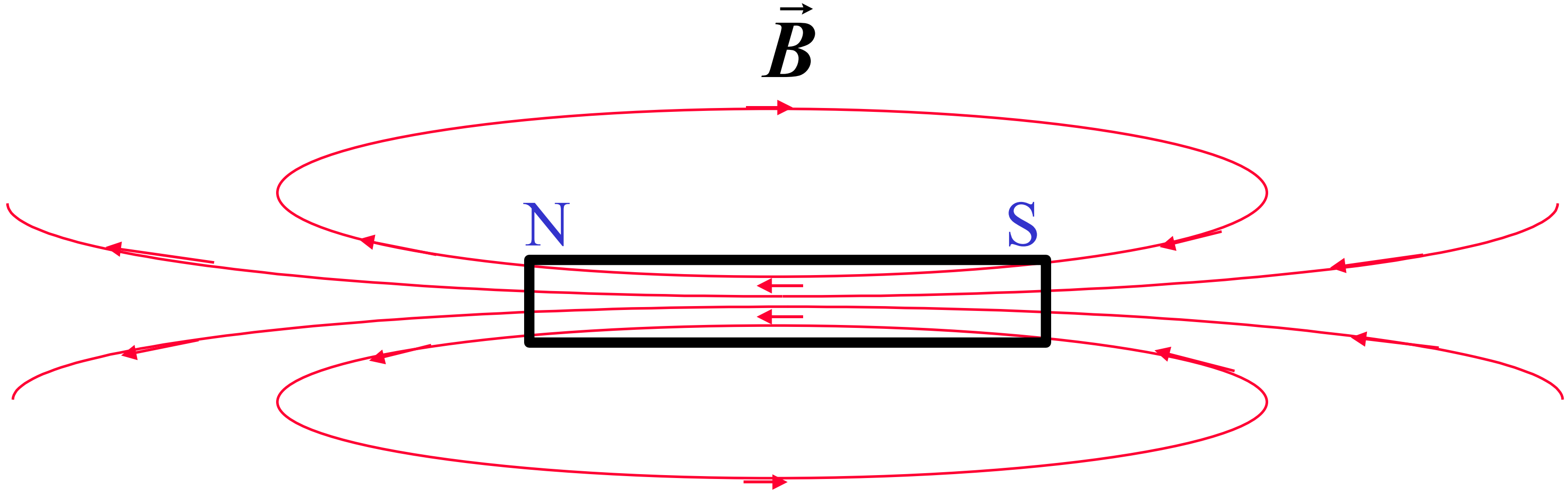
  a **magnetic field.**
- But we have not been able, so far, to isolate a magnetic monopole (the equivalent of an electric charge).
- We describe magnets as having two **magnetic poles**:

  North (**N**) and South (**S**).

What Do We Know About Permanent Magnets?

- They always have two poles.
- Like poles repel, opposite poles attract.
  - i.e. there are magnetic forces and ***fields!***
- They also attract un-magnetized ferromagnetic materials.
- We can map out the field of a magnet using either a small magnet or small magnetic materials....

Field of a Permanent Magnet



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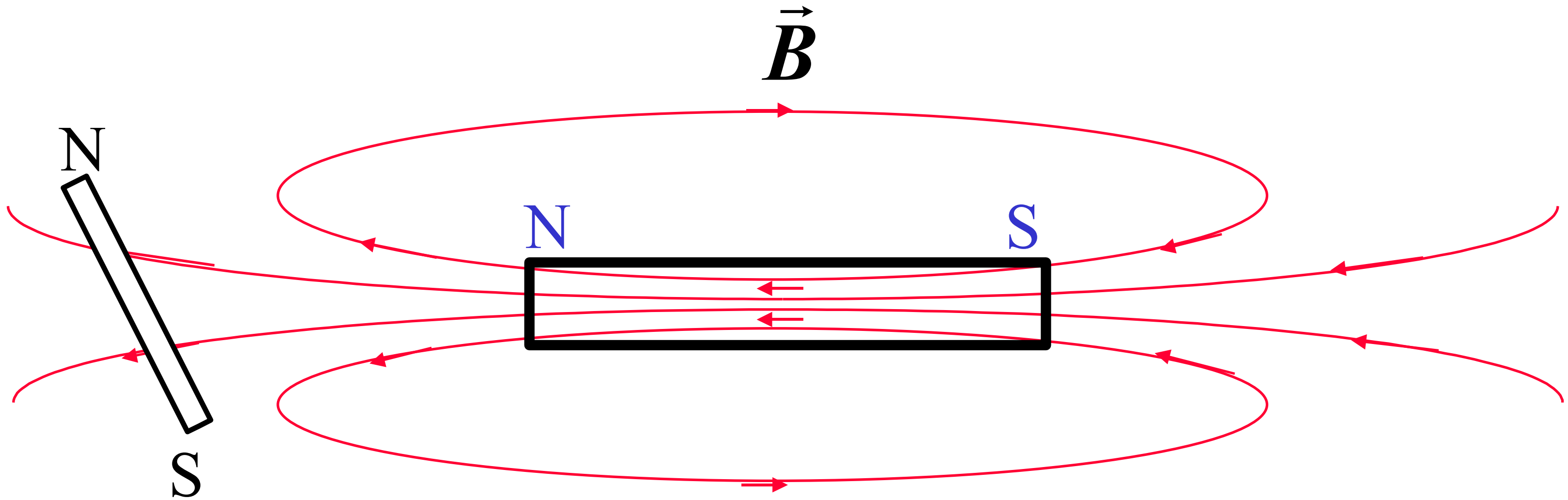

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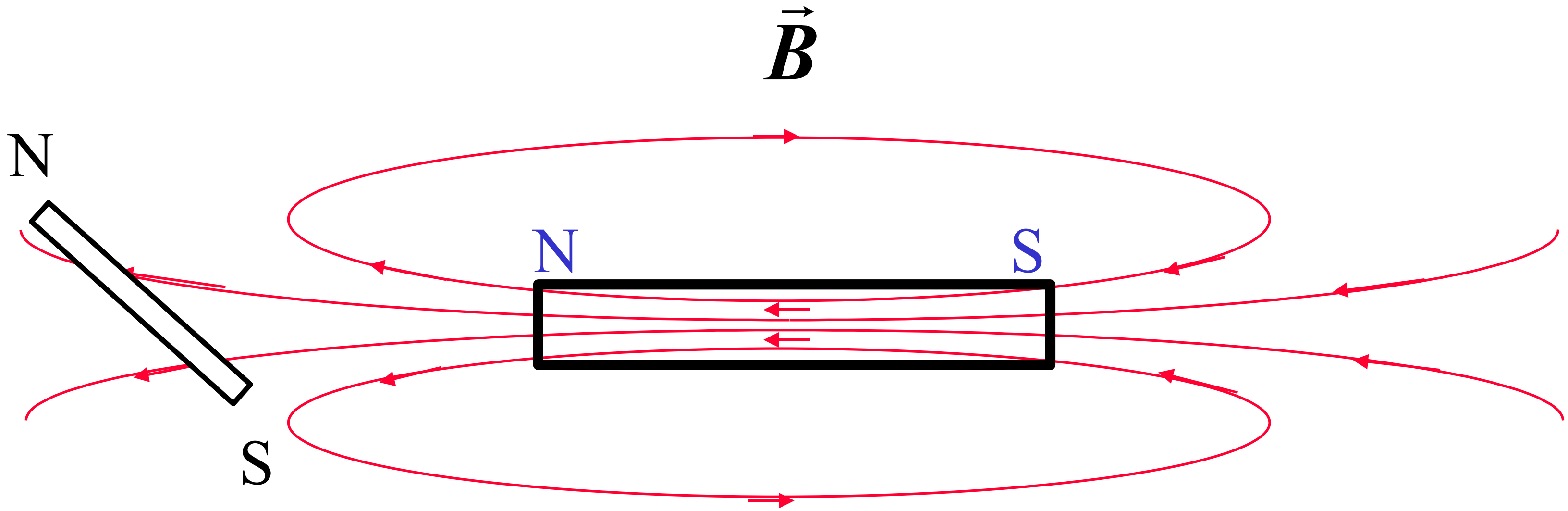
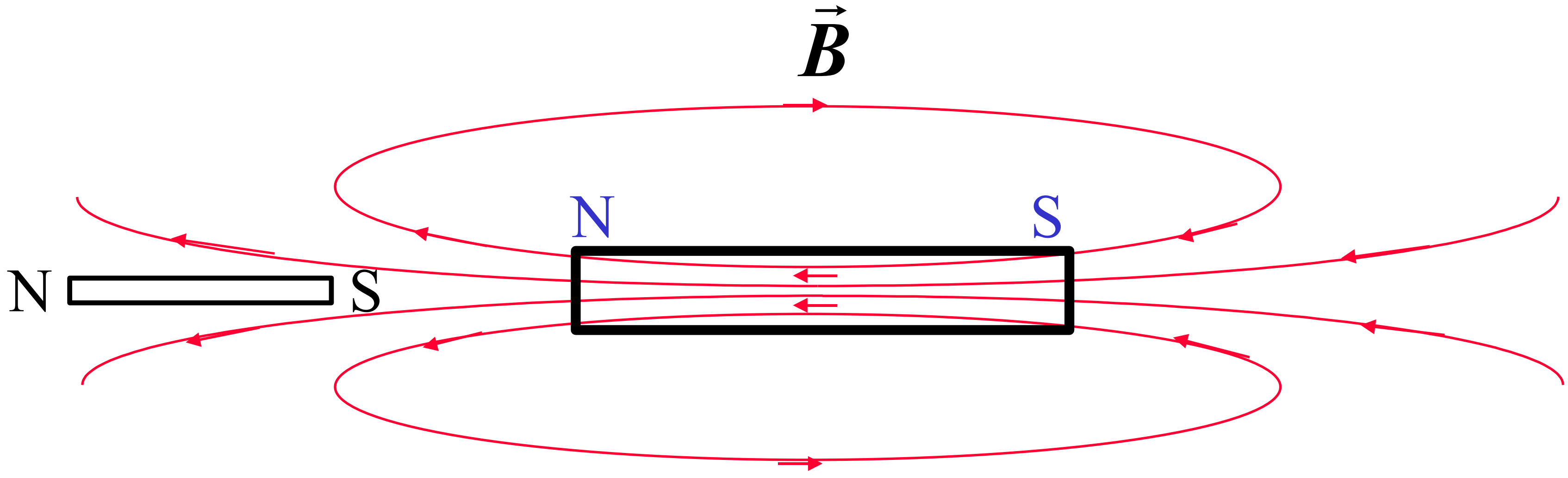


The south pole of the small bar magnet is attracted towards the north pole of the big magnet.

Also, the small bar magnet (a magnetic dipole) wants to align with the **B**-field.

The field **attracts** and **exerts a torque** on the small magnet.

Field of a Permanent Magnet



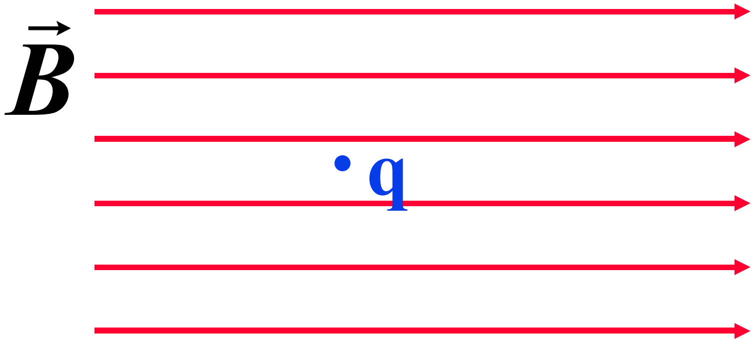
The bar magnet (a magnetic dipole) wants to align with the B-field.

The field exerts a <u>torque</u> on the dipole

Magnetism

- The origin of magnetism lies in moving electric charges. Moving (or rotating) charges generate magnetic fields.
- An electric current generates a magnetic field.
- A magnetic field will exert a force on a moving charge.
- A magnetic field will exert a force on a conductor that carries an electric current.

What Force Does a Magnetic Field Exert on Charges?



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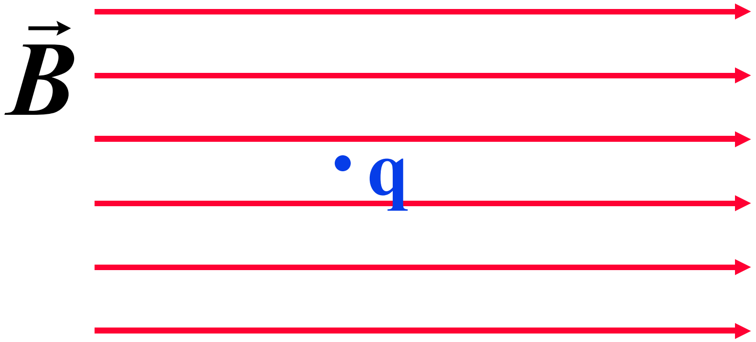

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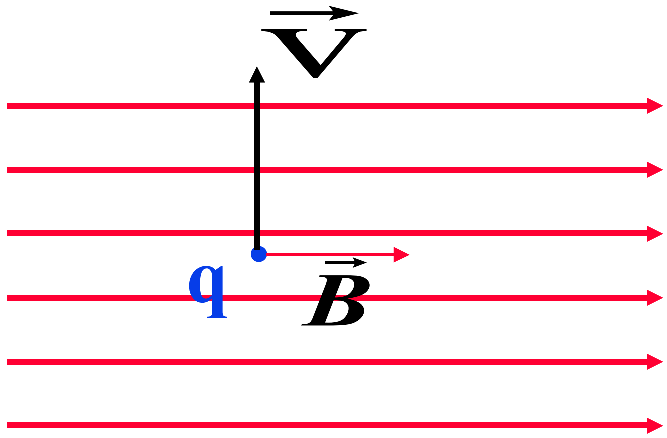

- If the charge is moving, there is a force on the charge, ***perpendicular*** to both <u>**v**</u> and <u>**B**</u>.

$$\mathbf{F} = q\,\mathbf{v} \times \mathbf{B}$$

Force on a Charge in a Magnetic Field

- As we saw, force is perpendicular to both **v** and **B**.
- The force is also largest for **v** perpendicular to **B**, smallest for **v** parallel to **B**.

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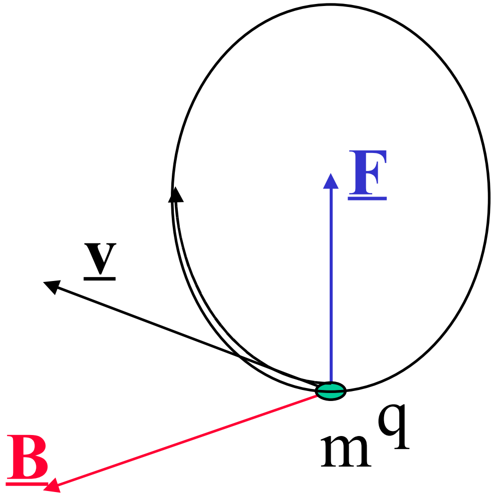
or:

$$F = qvB\sin\theta$$



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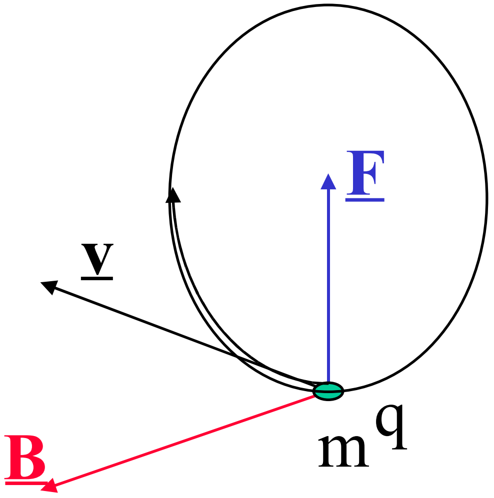

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(Note: 1 Tesla = 10,000 Gauss)

The Magnetic Force is *Different* From the Electric Force.

Whereas the electric force acts in the same direction as the field:

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The magnetic force acts in a direction orthogonal to the field:

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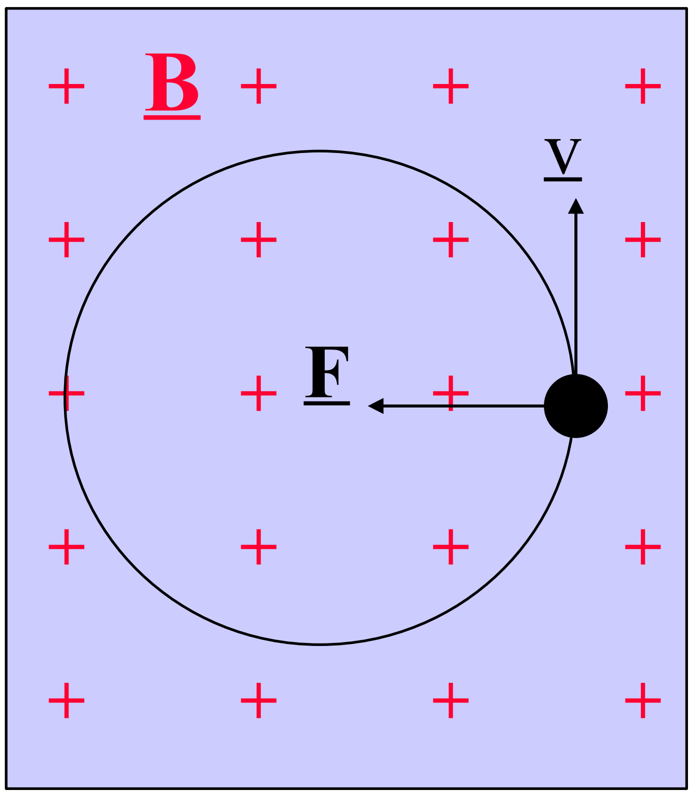
Trajectory of Charged Particles in a Magnetic Field

(B field points *into* plane of paper.)



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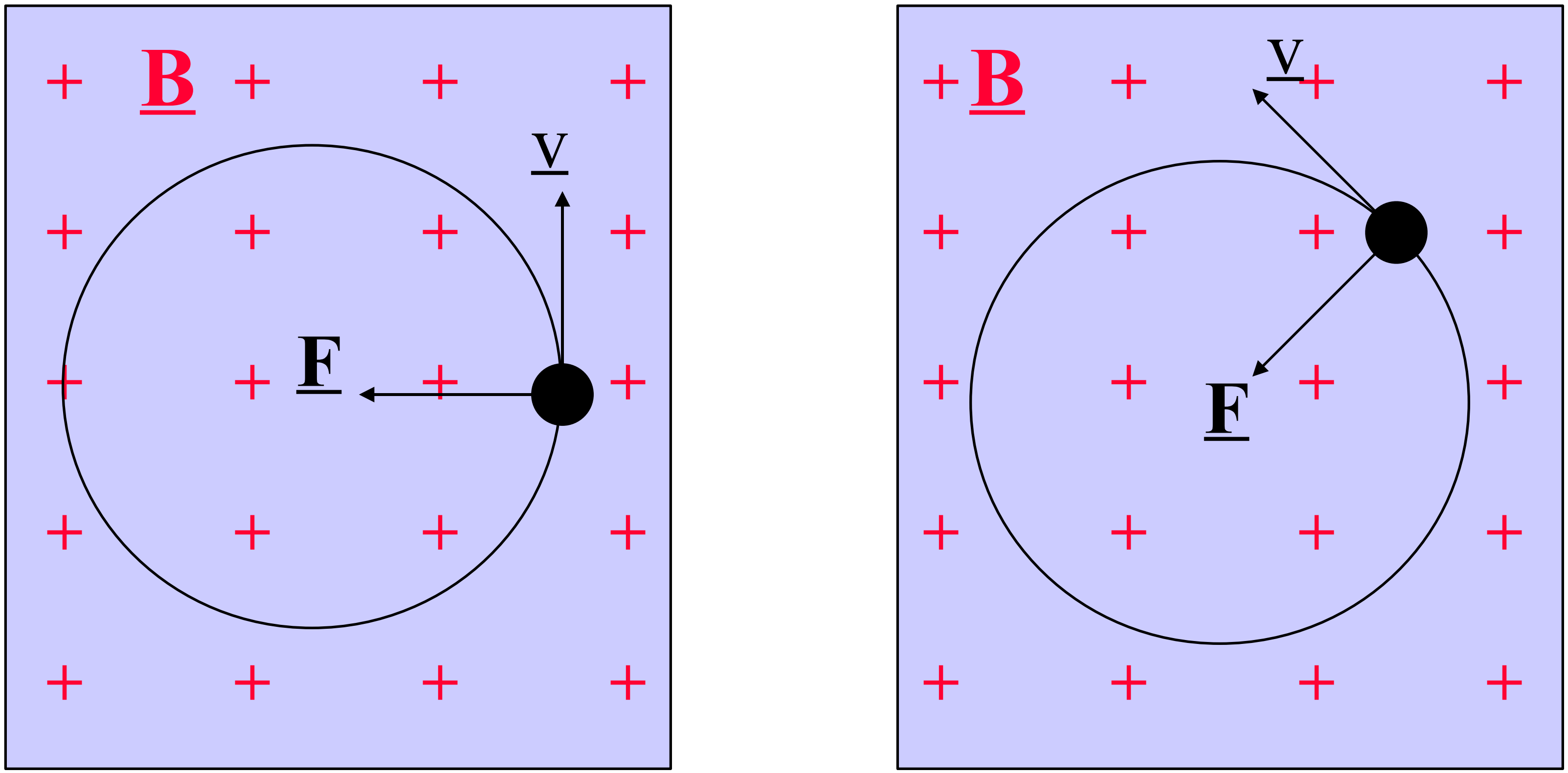

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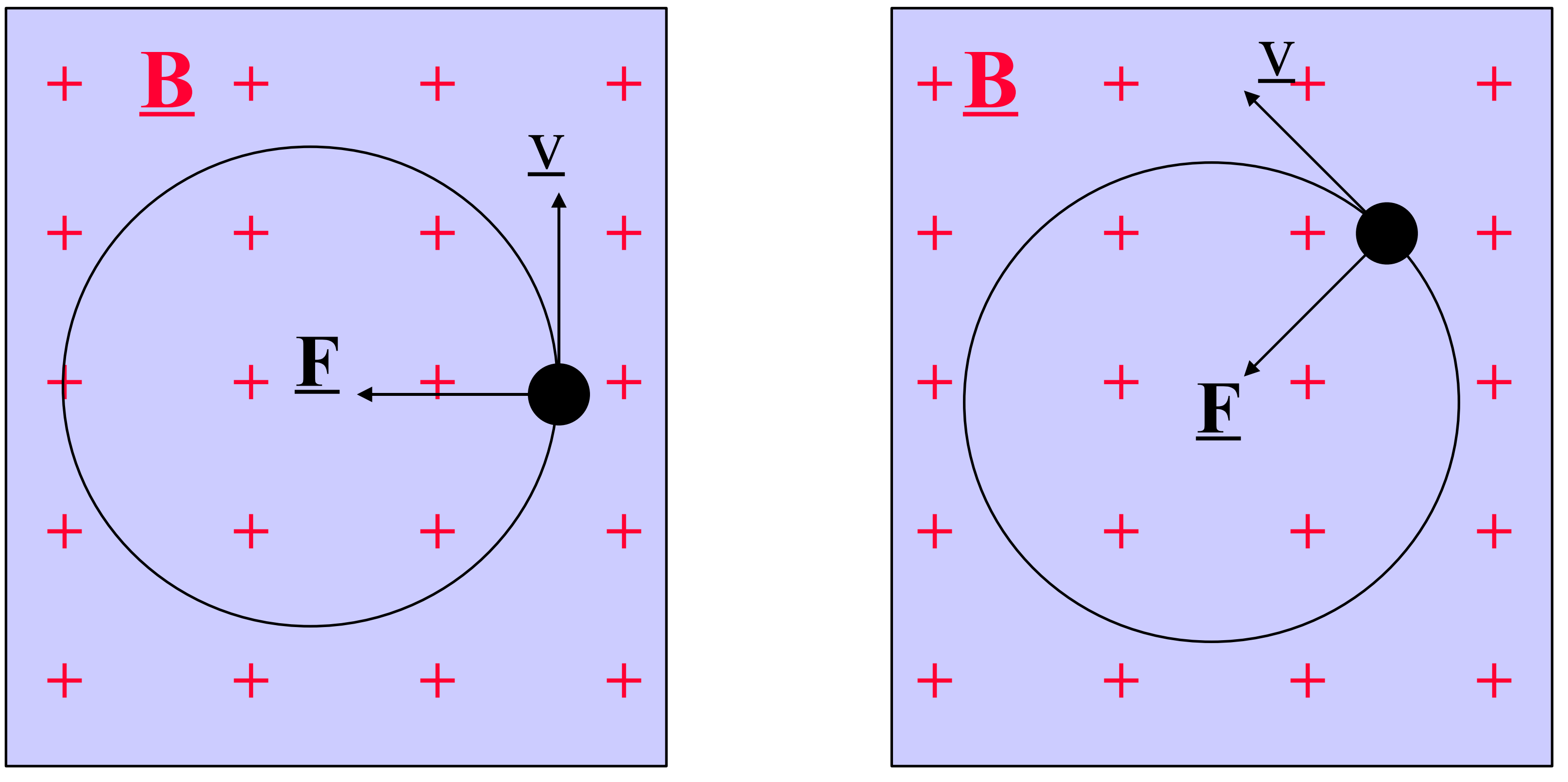
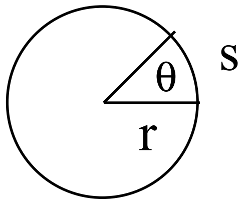


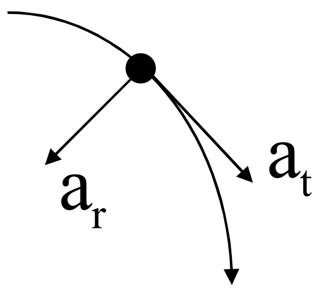
Magnetic Force is a ***centripetal*** force

Rotational Motion



$\theta = s / r \Rightarrow s = \theta\, r \Rightarrow ds/dt = d\theta/dt\ r \Rightarrow v = \omega\, r$

θ = angle, ω = angular speed, α = angular acceleration

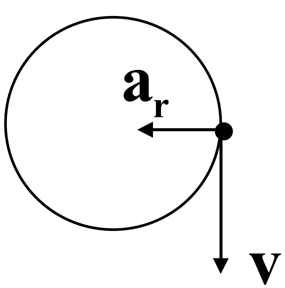


$a_t = r\,\alpha$ tangential acceleration

$a_r = v^2 / r$ radial acceleration

The radial acceleration changes the direction of motion,
while the tangential acceleration changes the speed.

Uniform Circular Motion



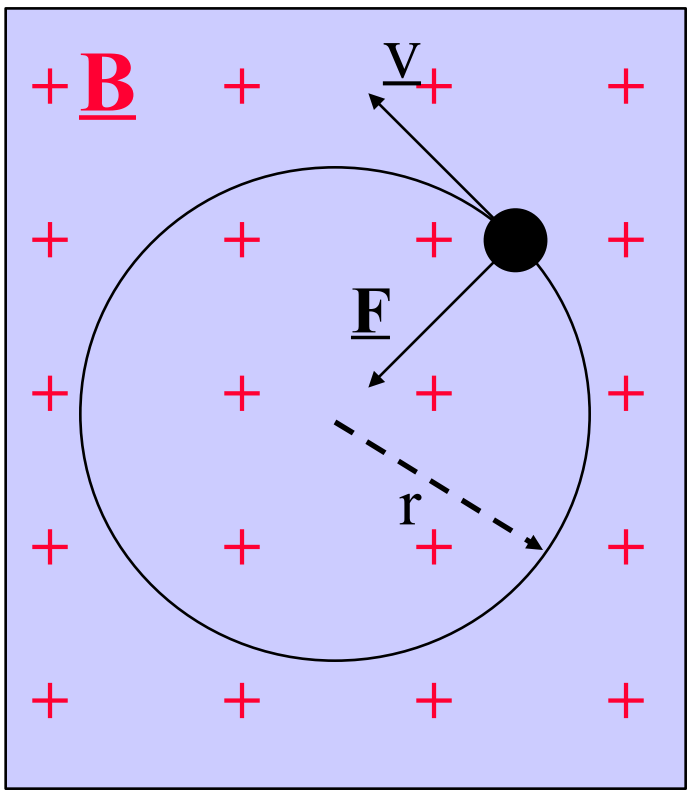
ω = constant $\Rightarrow$ v and a_r constant but direction changes

$a_r = v^2/r = \omega^2\, r$

$KE = ½\, mv^2 = ½\, mw^2r^2$

$F = ma_r = mv^2/r = m\omega^2 r$

Radius of Charged Particle Orbit in a Magnetic Field



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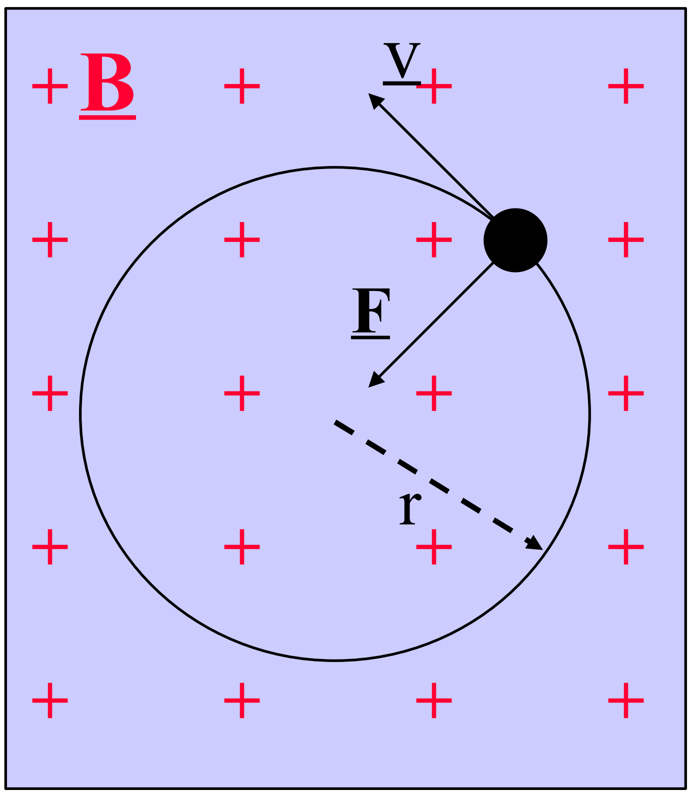

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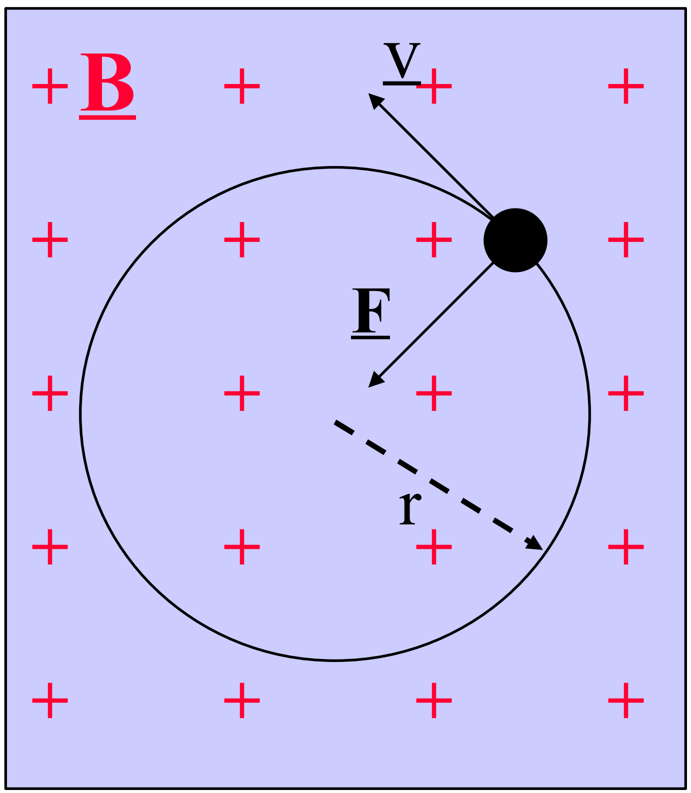

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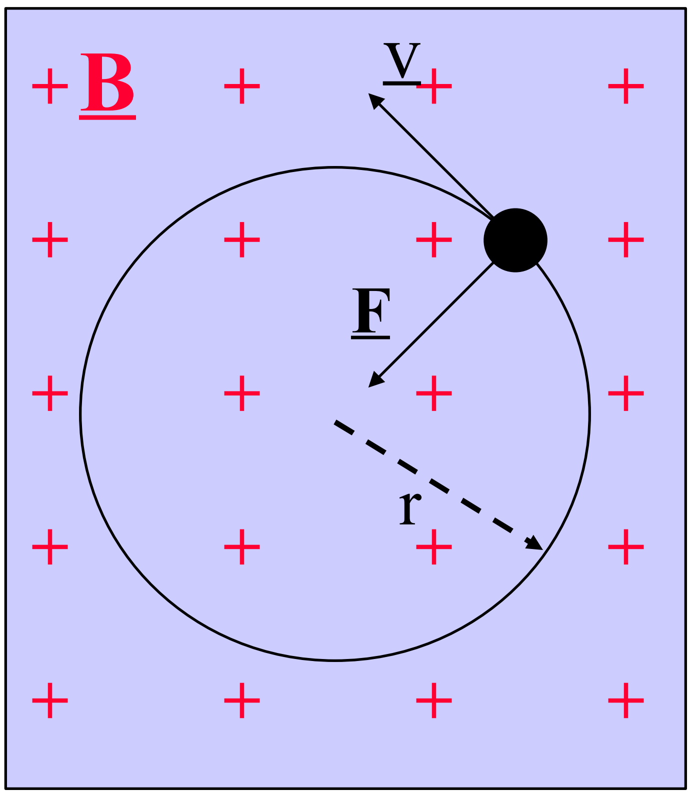

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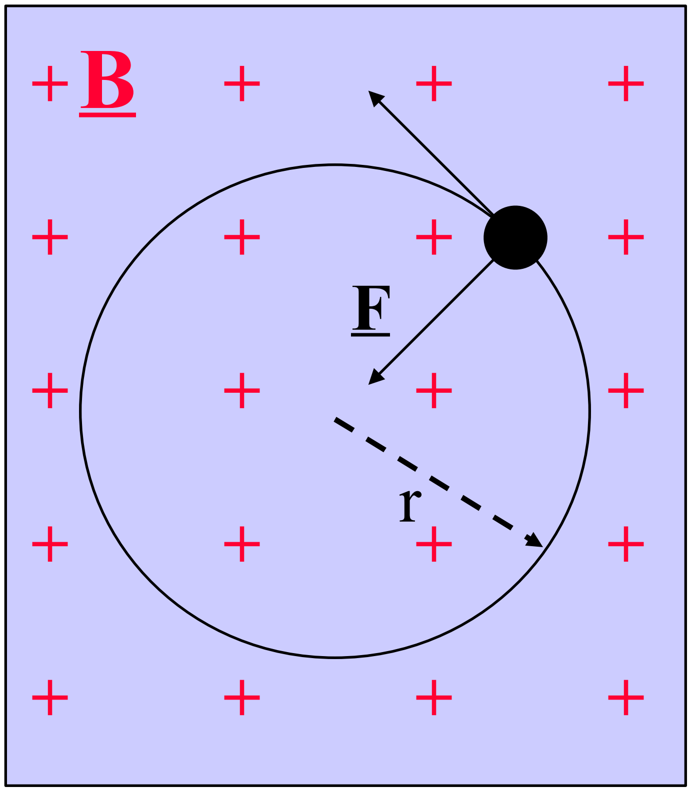
Note: as $\vec{F} \perp \vec{\text{v}}$, the magnetic force does no work!

Cyclotron Frequency



The time taken to complete one orbit is:

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v}\frac{mv}{qB}$$

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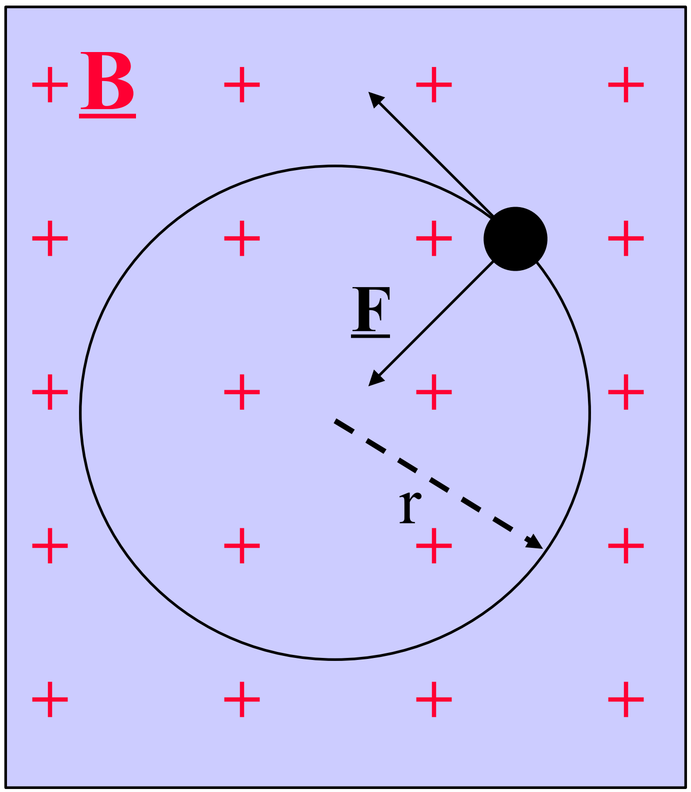

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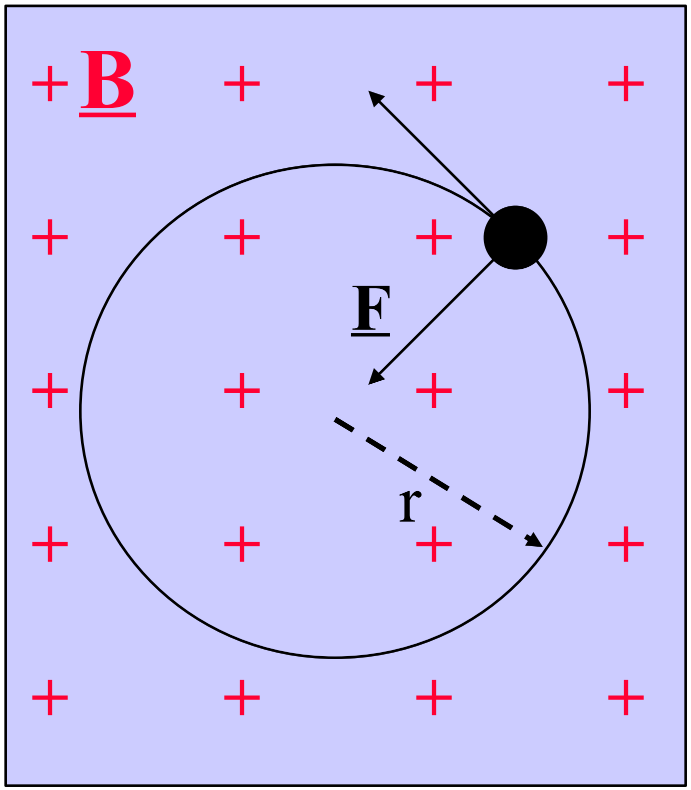

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- known as the "cyclotron frequency"

$$T = 2\pi/\omega = 1/f \Rightarrow f = \omega/2\pi$$

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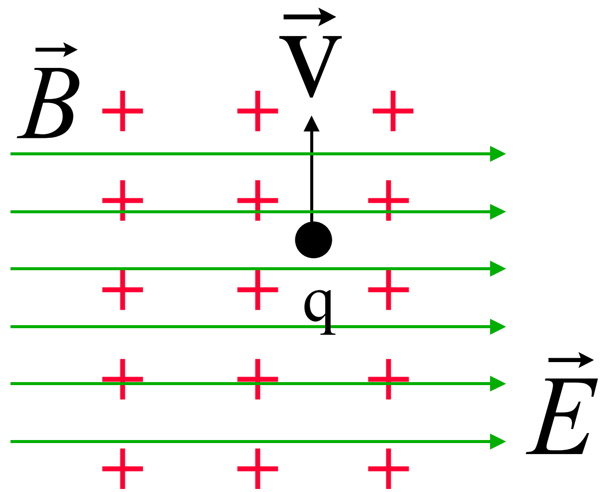

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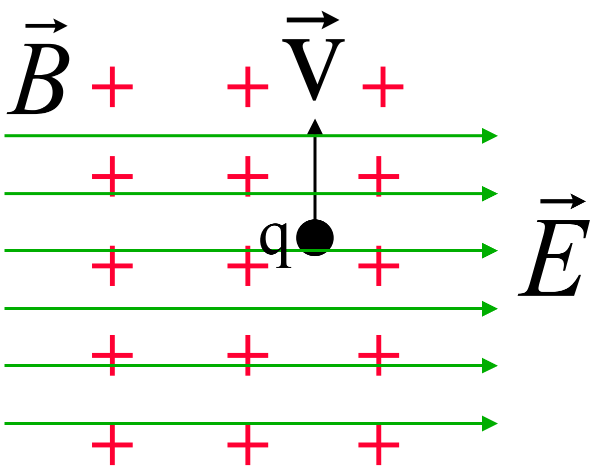
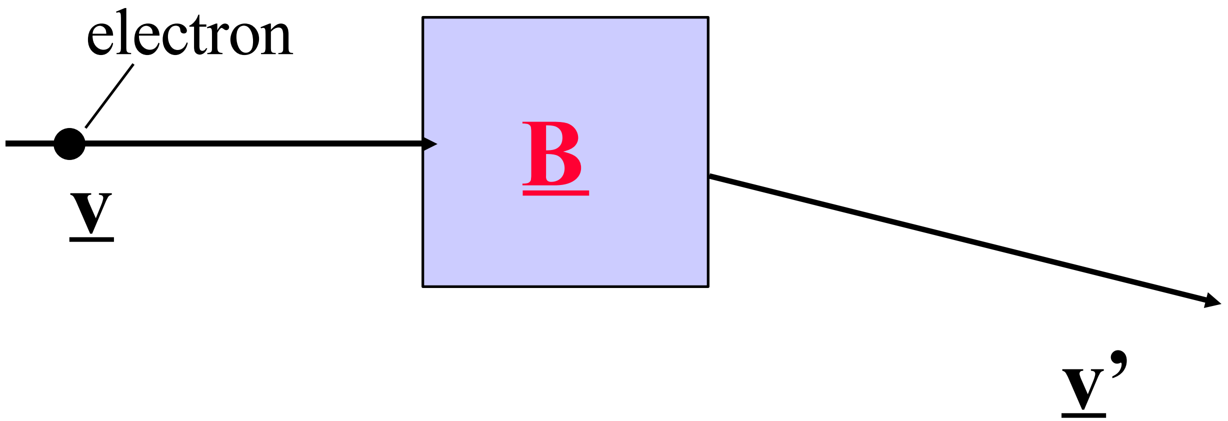


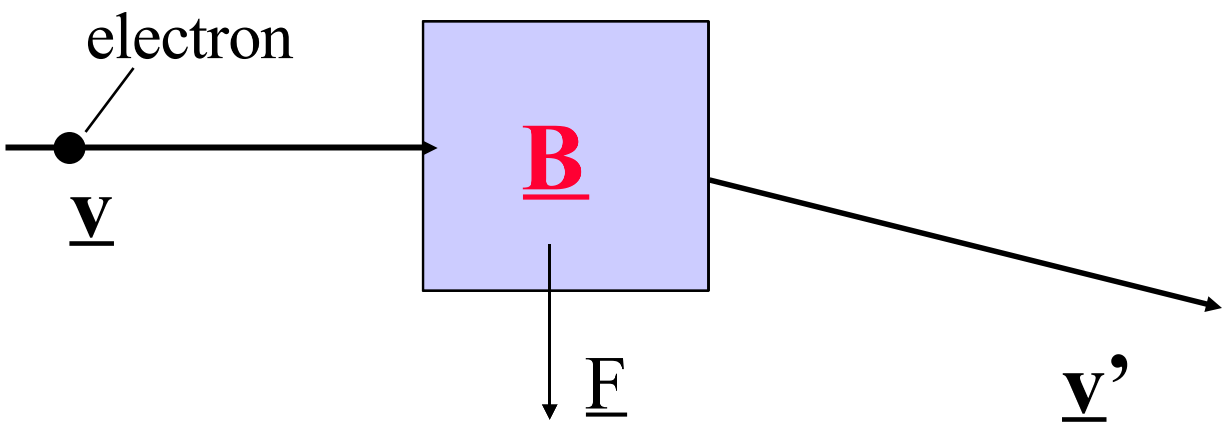
- Here, $\vec{F}_E$ and $\vec{F}_B$ point in opposite directions

Exercise



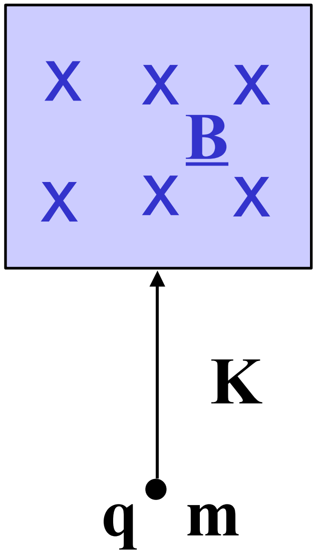
- In what direction does the magnetic field point?
- Which is bigger, **v** or **v'** ?

Exercise: answer



- In what direction does the magnetic field point ?
  Into the page [$\underline{F} = -e\, \underline{v} \times \underline{B}$]
- Which is bigger, **v** or **v'** ?
  v = v' [B does no work on the electron, $\underline{F} \perp \underline{v}$]

What is the orbital radius of a charged particle (charge q, mass m) having kinetic energy K, and moving at right angles to a magnetic field B, as shown below?.



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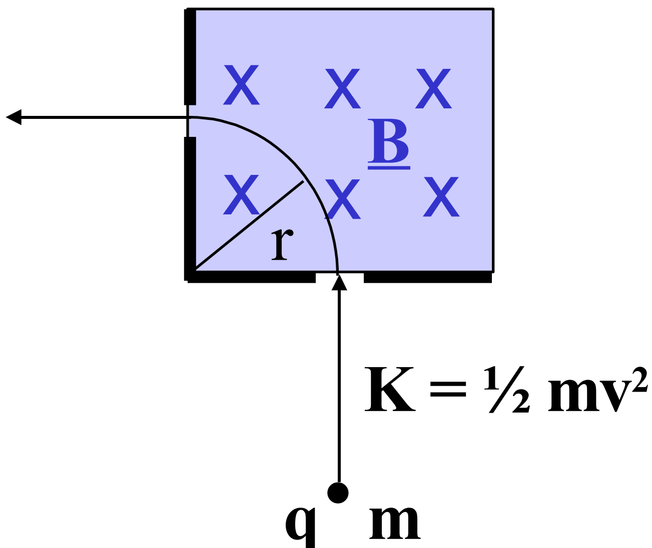
$$\underline{F} = q\,\underline{v} \times \underline{B} = m\,\underline{a} \quad \text{and} \quad a = v^2 / r$$



$$q\,v\,B = m\,v^2 / r$$

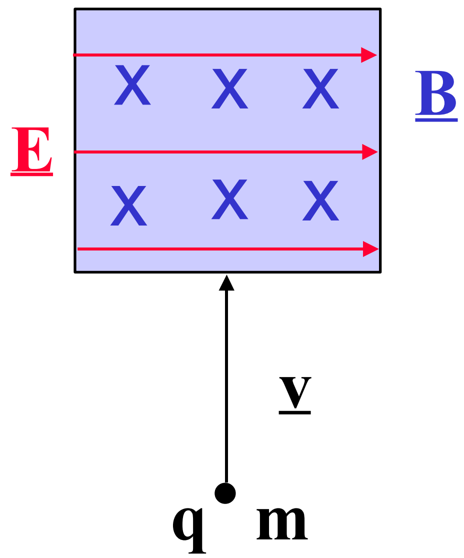
$$q\,B = m\,v / r \Rightarrow r\,q\,B = m\,v$$

$$r = m\,v / (q\,B)$$

$$r^2 = m^2\,v^2 / (q\,B)^2 \Rightarrow (1/2m)\,r^2 = \tfrac{1}{2}\,m\,v^2 / (q\,B)^2$$

$$(1/2m)\,r^2 = K / (q\,B)^2 \Rightarrow \boxed{r = [2mK]^{1/2} / (q\,B)}$$

What is the relation between the intensities of the electric and magnetic fields for the particle to move in a straight line ?.



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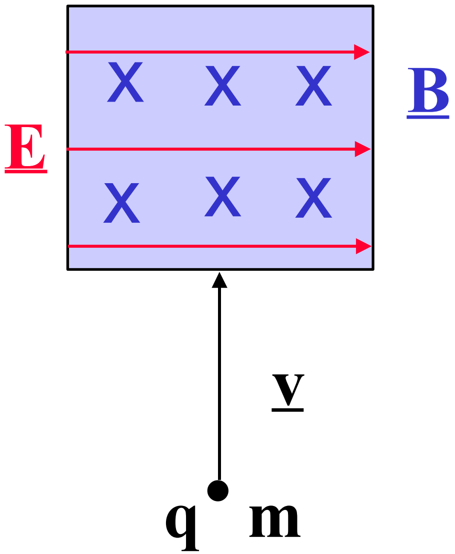
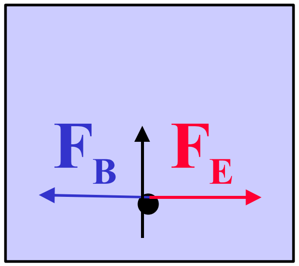


$$F_E = q\,E \quad \text{and} \quad F_B = q\,v\,B$$

If $F_E = F_B$ the particle will move following a straight line trajectory

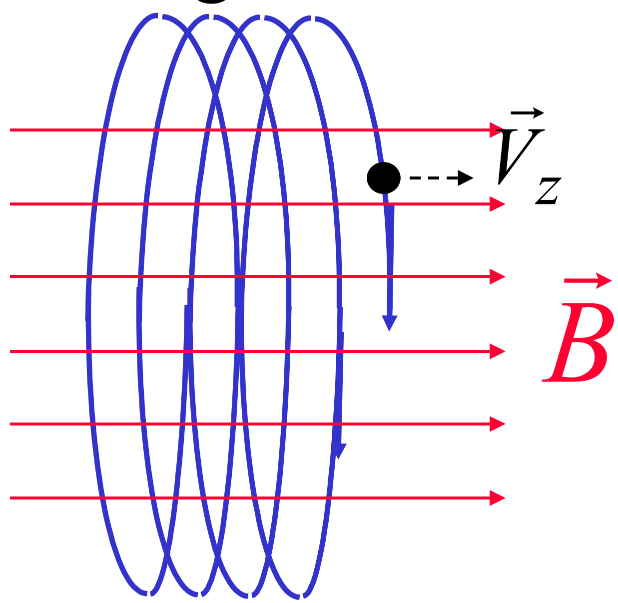
$$q\,E = q\,v\,B$$



$$\boxed{v = E / B}$$

Trajectory of Charged Particles in a Magnetic Field

What if the charged particle has a velocity component along B?



$\vec{V}_z$ unchanged

Circular motion in xy plane.