DC Electrical Circuits

Chapter 28

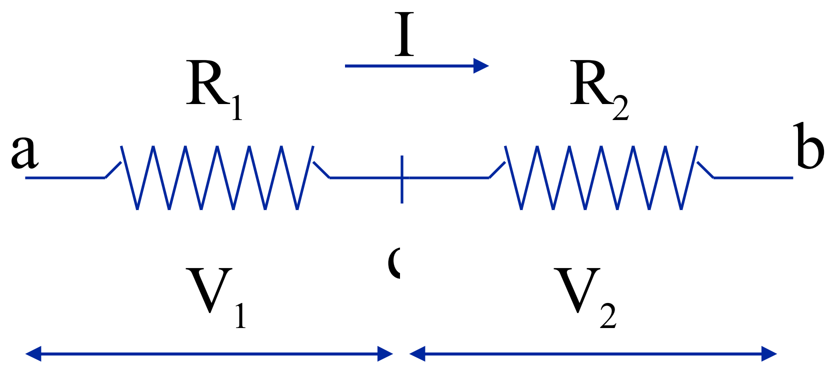
Electromotive Force

Potential Differences

Resistors in Parallel and Series

Circuits with Capacitors

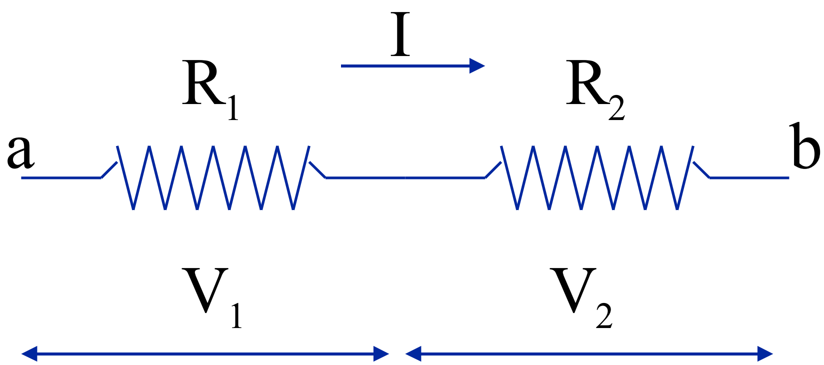
Resistors in Series



The pair of resistors, R_1 and R_2, can be replaced by a single equivalent resistor R; one which, given I, has the same total voltage drop as the original pair.

Note: the current I is the same, anywhere between a and b, but there is a voltage drop V_1 across R_1, and a voltage drop V_2 across R_2.

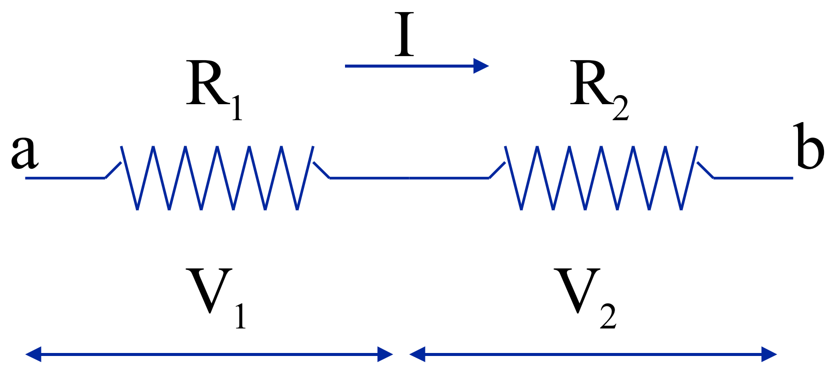
Resistors in Series



The pair of resistors can be replaced by a single equivalent resistor R_{eq}; one which, given I, has the same total voltage drop as the original pair.

$$V = V_1 + V_2 = I\,R_1 + I\,R_2$$

Resistors in Series

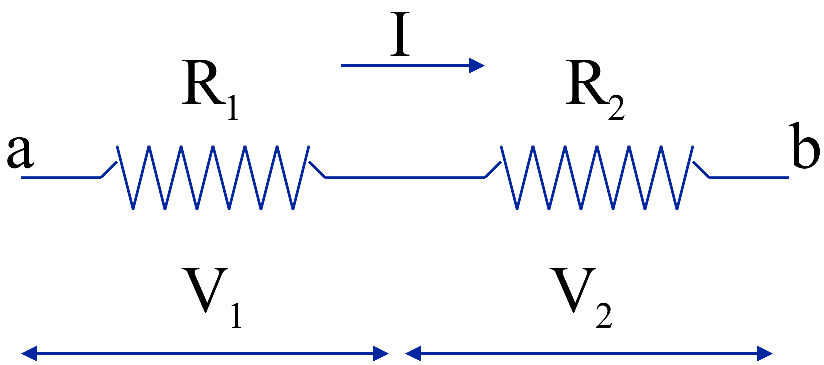


The pair of resistors can be replaced by a single equivalent resistor R_{eq}; one which, given I, has the same total voltage drop as the original pair.

$$V = V_1 + V_2 = I\,R_1 + I\,R_2$$

We want to write this as $V = I\,R_{eq}$

Resistors in Series



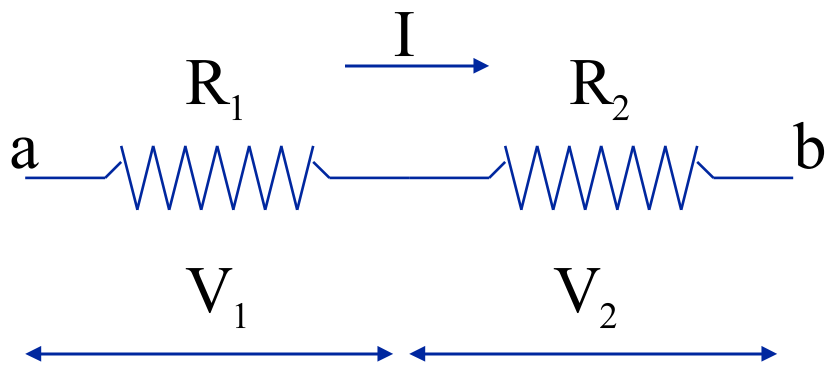
The pair of resistors can be replaced by a single equivalent resistor R_{eq}; one which, given I, has the same total voltage drop as the original pair.

$$V = V_1 + V_2 = I\,R_1 + I\,R_2$$

We want to write this as $V = I\,R_{eq}$

hence $R_{eq} = R_1 + R_2$

Resistors in Series

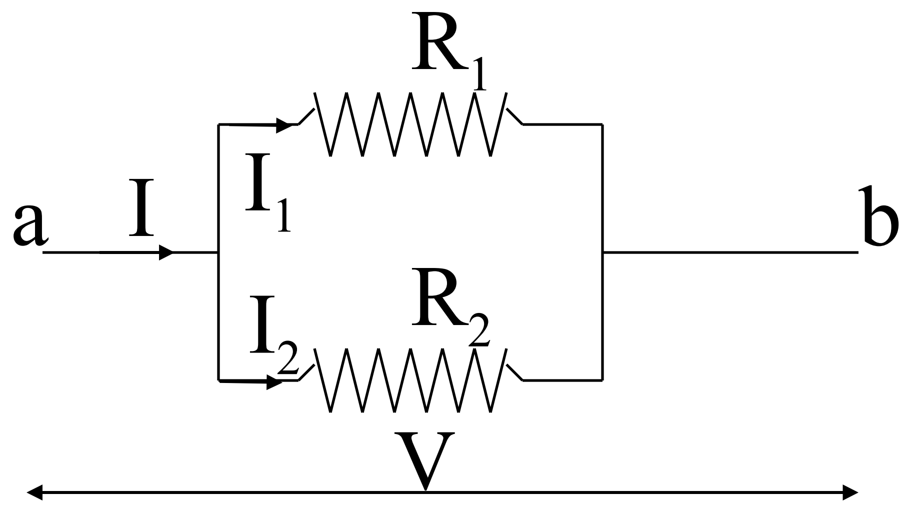


What is the voltage drop across each resistor ?

$V_1 = I\,R_1$ but $I = V / (R_1 + R_2)$ $\Rightarrow$ $\boxed{V_1 = V\,R_1 / (R_1 + R_2)}$

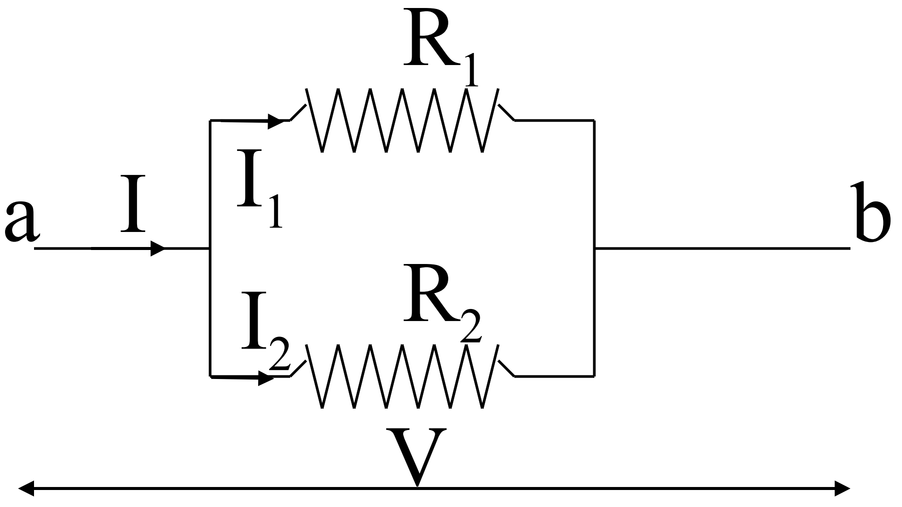
$V_2 = I\,R_2$ but $I = V / (R_1 + R_2)$ $\Rightarrow$ $\boxed{V_2 = V\,R_2 / (R_1 + R_2)}$

Resistors in Parallel



Again find the equivalent single resistor which has the same V if I is given.

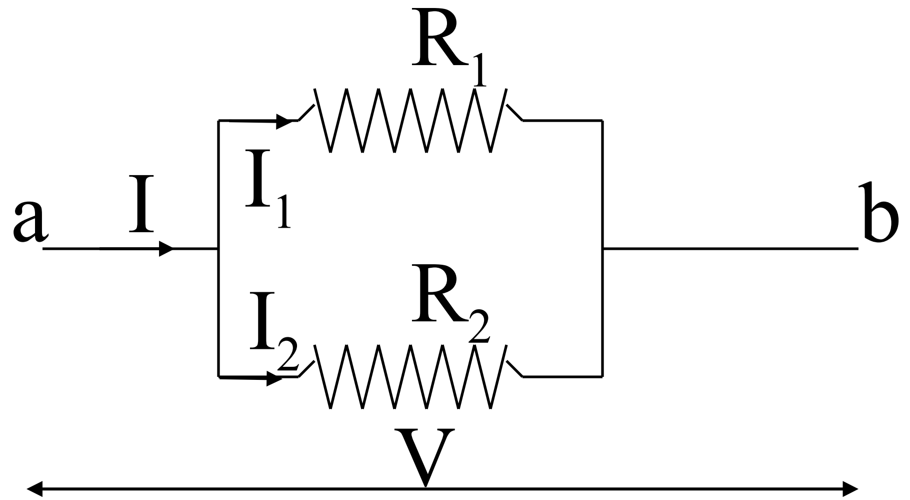
Resistors in Parallel



Again find the equivalent single resistor which has the same V if I is given. Here the total I splits:

$$I = I_1 + I_2 = V / R_1 + V / R_2 = V(1/ R_1 + 1/ R_2)$$

Resistors in Parallel

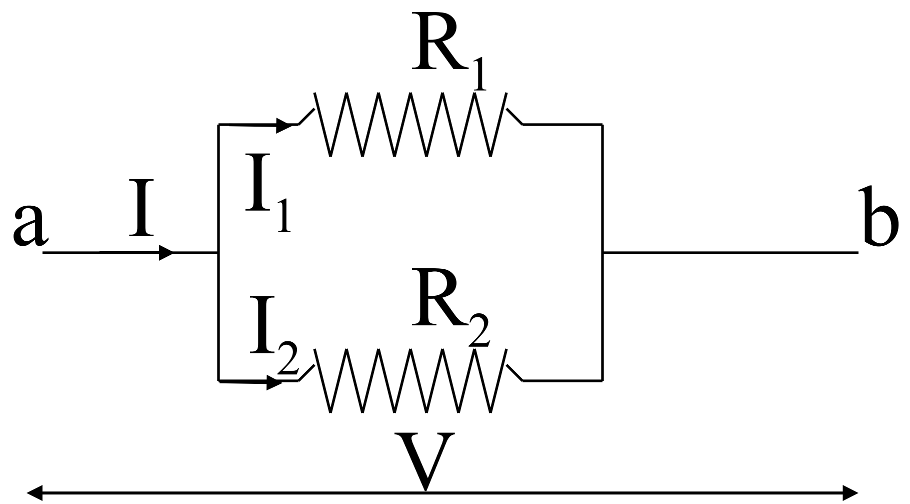


Again find the equivalent single resistor which has the same V if I is given. Here the total I splits:

$$I = I_1 + I_2 = V / R_1 + V / R_2 = V(1/ R_1 + 1/ R_2)$$

We want to write this as: $I = V / R_{eq}$

Resistors in Parallel



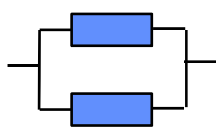
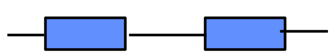
Again find the equivalent single resistor which has the same V if I is given. Here the total I splits:

$$I = I_1 + I_2 = V / R_1 + V / R_2 = V(1/ R_1 + 1/ R_2)$$

We want to write this as: $I = V / R_{eq}$

Hence $$1 / R_{eq} = 1/ R_1 + 1/ R_2$$

Parallel and Series

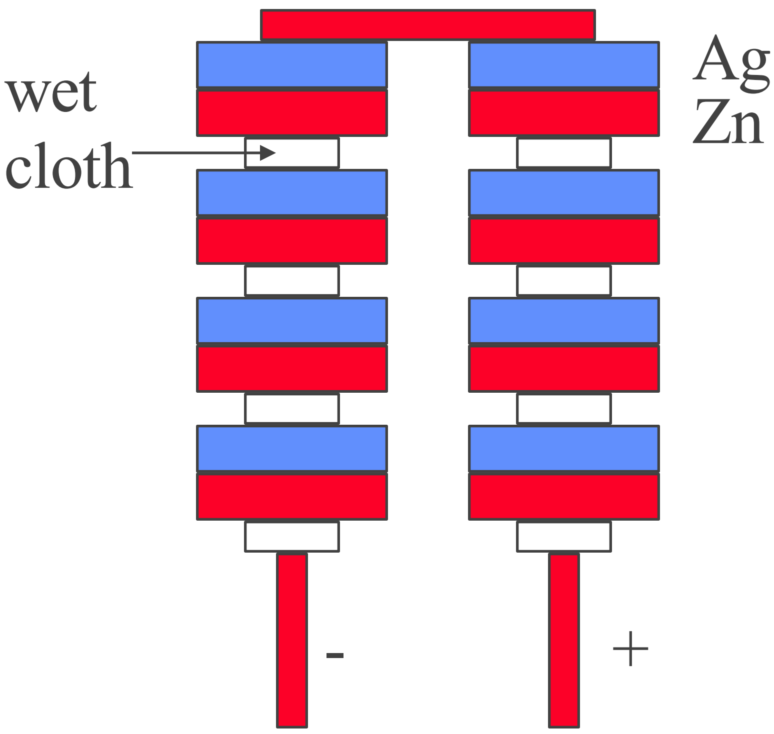
	Resistors	Capacitors
Parallel 	$1/R=1/R_1+1/R_2$	$C=C_1+C_2$
Series 	$R=R_1+R_2$	$1/C=1/C_1+1/C_2$

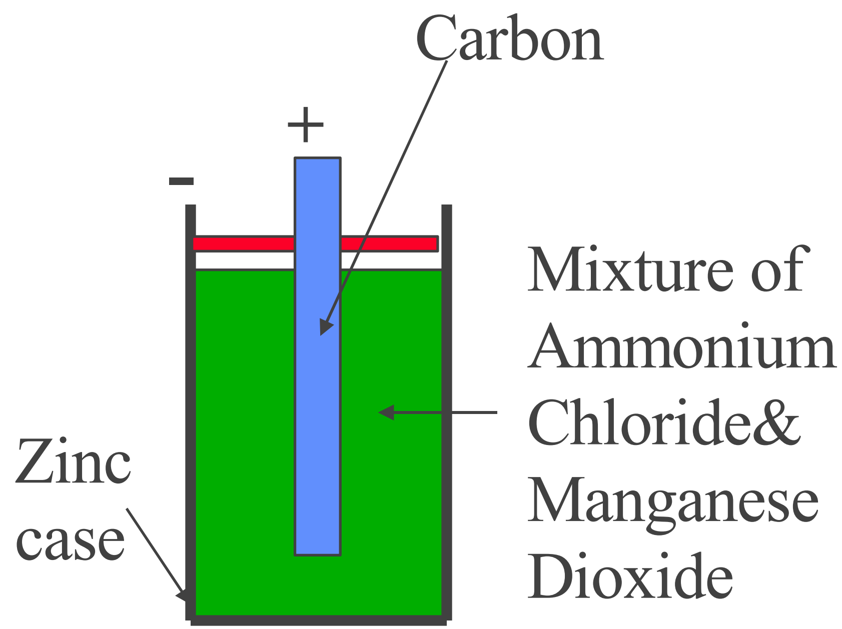
Batteries and Generators

- Current is produced by applying a potential difference across a conductor (I=V/R) [This is not equilibrium so there is an electric field inside the conductor].
- This potential difference is set up by some source, such as a battery or generator [that generates charges, from some other type of energy, *i.e.* chemical, solar, mechanical].
- Conventionally an "applied voltage" is given the symbol ***E*** (units: volts).
- For historical reasons, this applied voltage is often called the "electromotive force" (**emf**) [even though it's not a force].

The Voltaic Pile

Volta's original battery

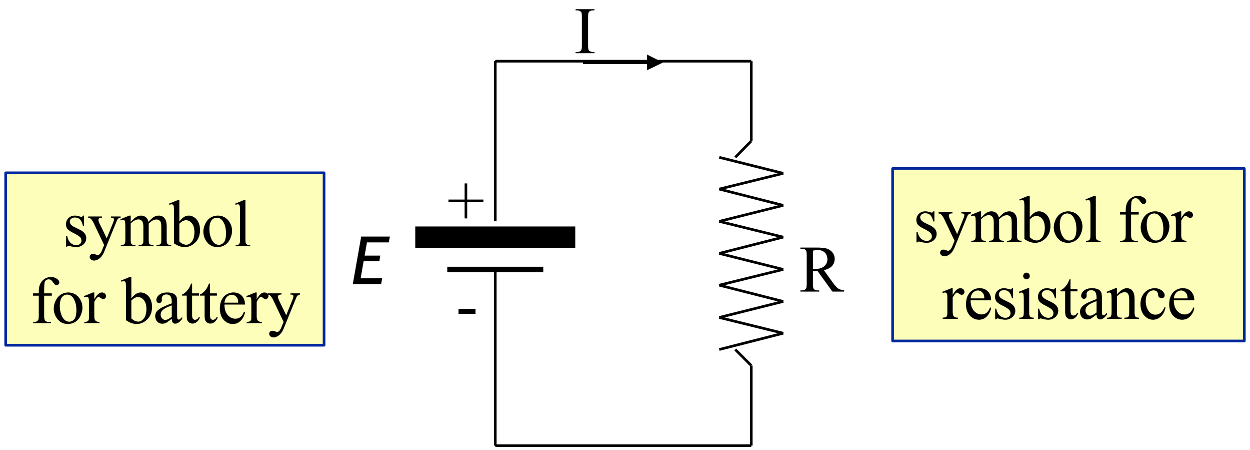




electrical converter...

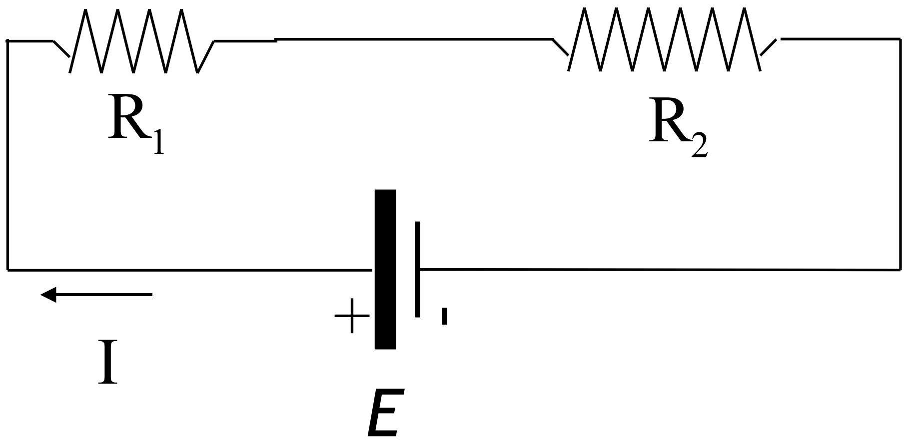
.....converts chemical energy to electrical energy

Electrical Description of a Battery



- **A battery does work on positive charges in moving them to higher potential (inside the battery).**
- **The EMF *E*, most precisely, is the work per unit charge exerted to move the charges "uphill" (to the + terminal, inside)**
- **... but you can just think of it as an "applied voltage."**
- **Current will flow, in the external circuit, from the + terminal, to the – terminal, of the battery.**

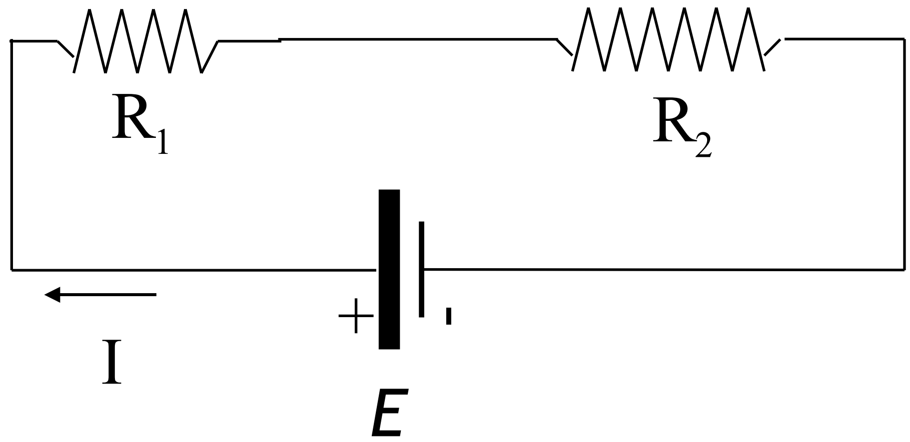
Resistors in Series



Given $R_{eq} = R_1 + R_2$, the current is $I = E/(R_1 + R_2)$

One can then work backwards to get the voltage across each resistor:

Resistors in Series

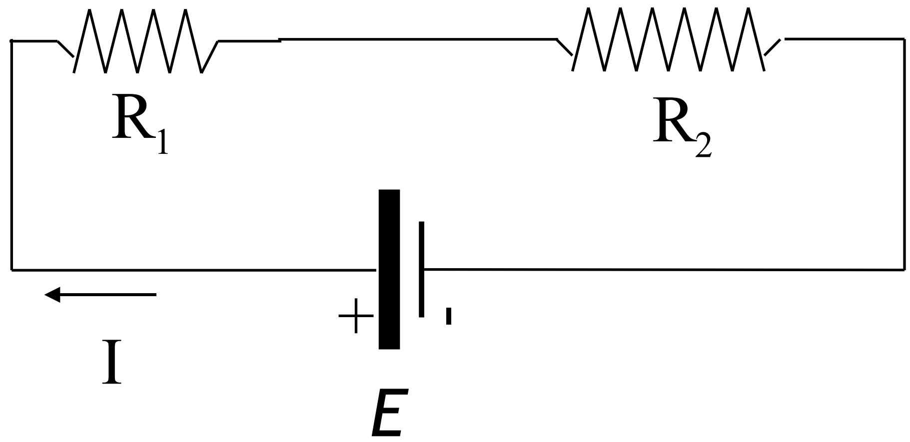


Given $R_{eq} = R_1 + R_2$, the current is $I = \mathcal{E}/(R_1 + R_2)$

One can then work backwards to get the voltage across each resistor:

$$V_1 = IR_1 = \mathcal{E}\frac{R_1}{R_1 + R_2}$$

The Loop Method



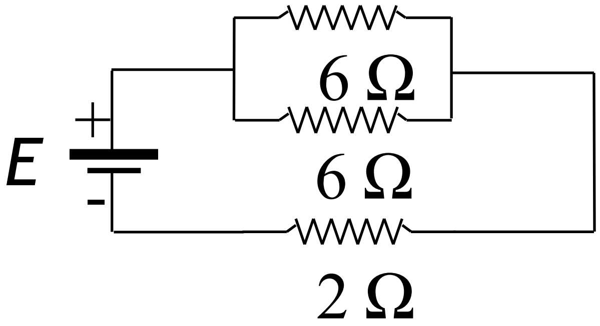
Go around the circuit in one direction.
If you pass a voltage source from – to +,
⇒ the voltage increases by *E* (or V).

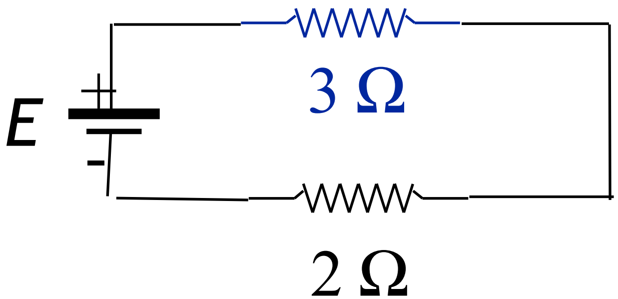
As you pass a resistor the voltage decreases by $V = I R$.

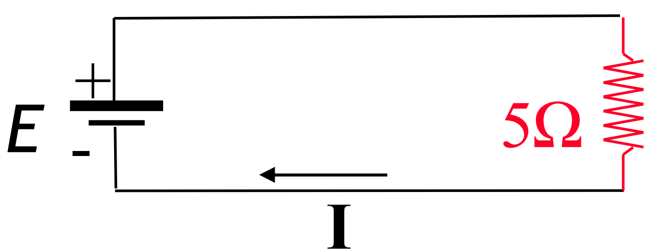
The total change in voltage after a complete loop is zero.

Analyzing Resistor Networks

1. Replace resistors step by step.

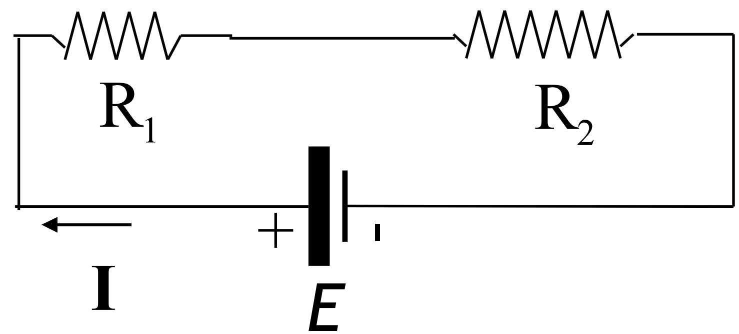






$$I = E / R$$

2. The loop method



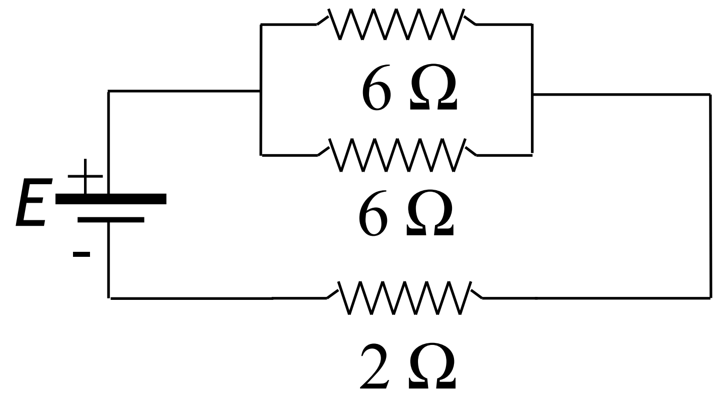
$$E - IR_1 - IR_2 = 0$$

$$E = I\,(R_1 + R_2)$$

$$I = E / (R_1 + R_2)$$

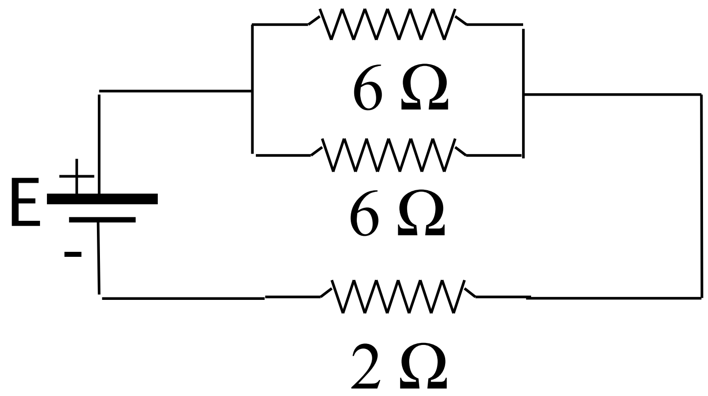
Analyzing Resistor Networks

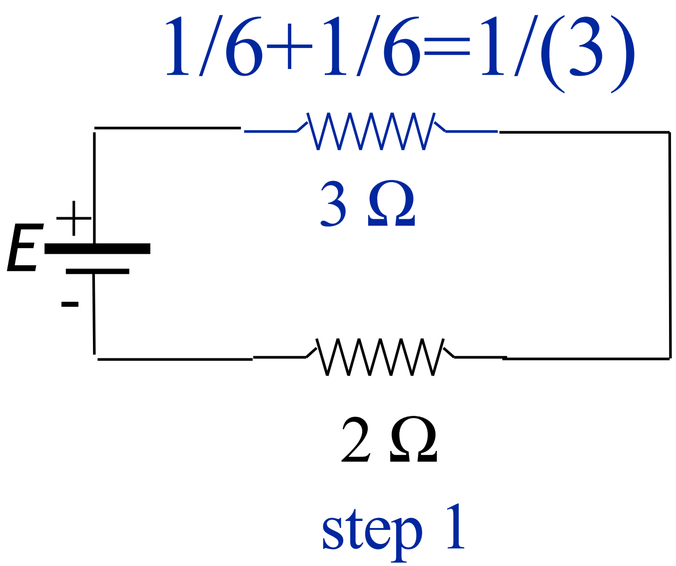
Often you can replace sets of resistors step by step.



Analyzing Resistor Networks

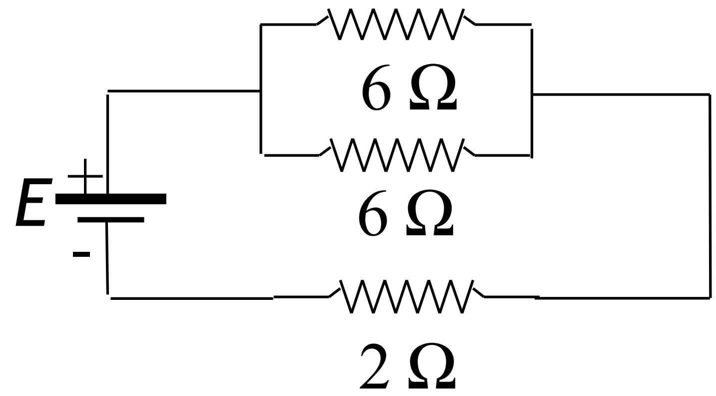
Often you can replace sets of resistors step by step.



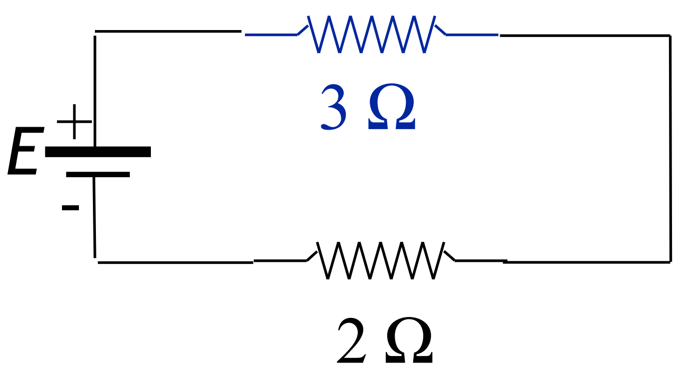


Analyzing Resistor Networks

Often you can replace sets of resistors step by step.



$$1/6+1/6=1/(3)$$

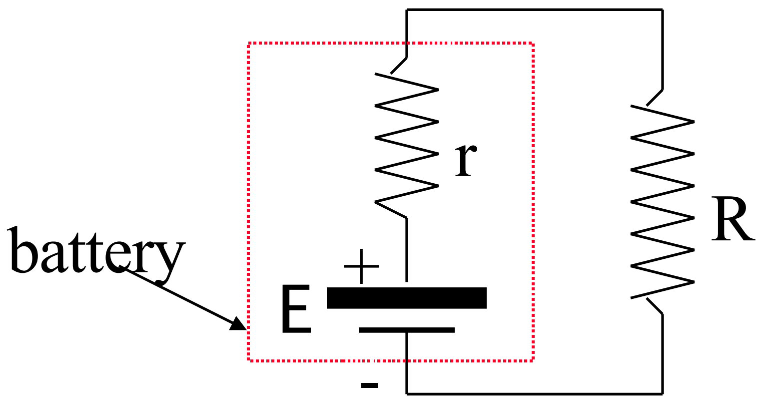


step 1

$$3 + 2 = 5$$

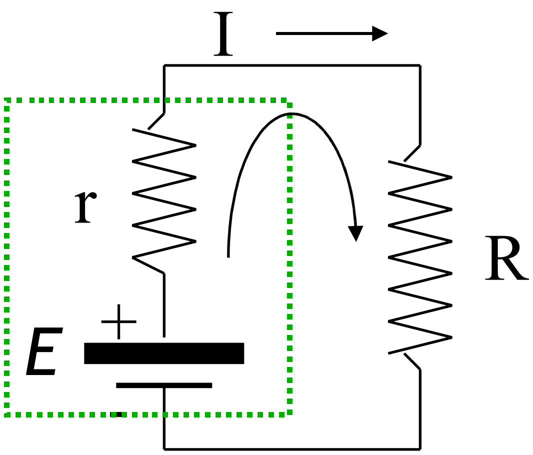
step 2

Internal Resistance of a Battery



- One important point: batteries actually have an
  internal resistance r
- Often we neglect this, but sometimes it is significant.

Effect of Internal Resistance Analyzed by the Loop Method



Start at any point in the circuit. Go around the circuit in a loop. Add up (subtract) the potential differences across each element (keep the signs straight!).

$$E - Ir - IR = 0 \text{ (using V=IR)} \Rightarrow I = E/(R + r)$$

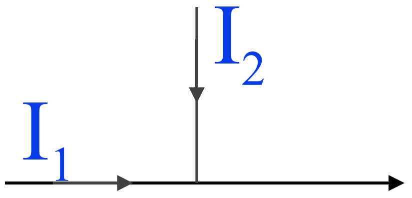
$$V_R = I\,R = E R/(R + r) = E/[1 + (r/R)]$$

$$\text{if } r << R \Rightarrow V_R = E$$

Kirchhoff's Laws

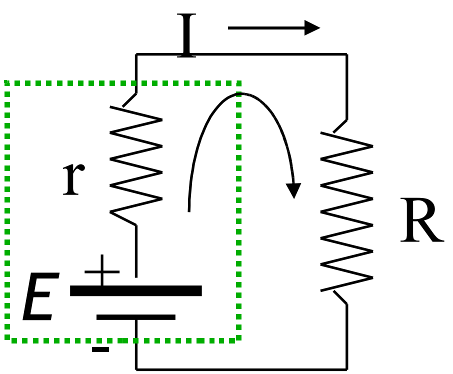
Kirchoff devised two laws that are universally applicable in circuit analysis:

1. At any circuit junction, currents entering must equal currents leaving.



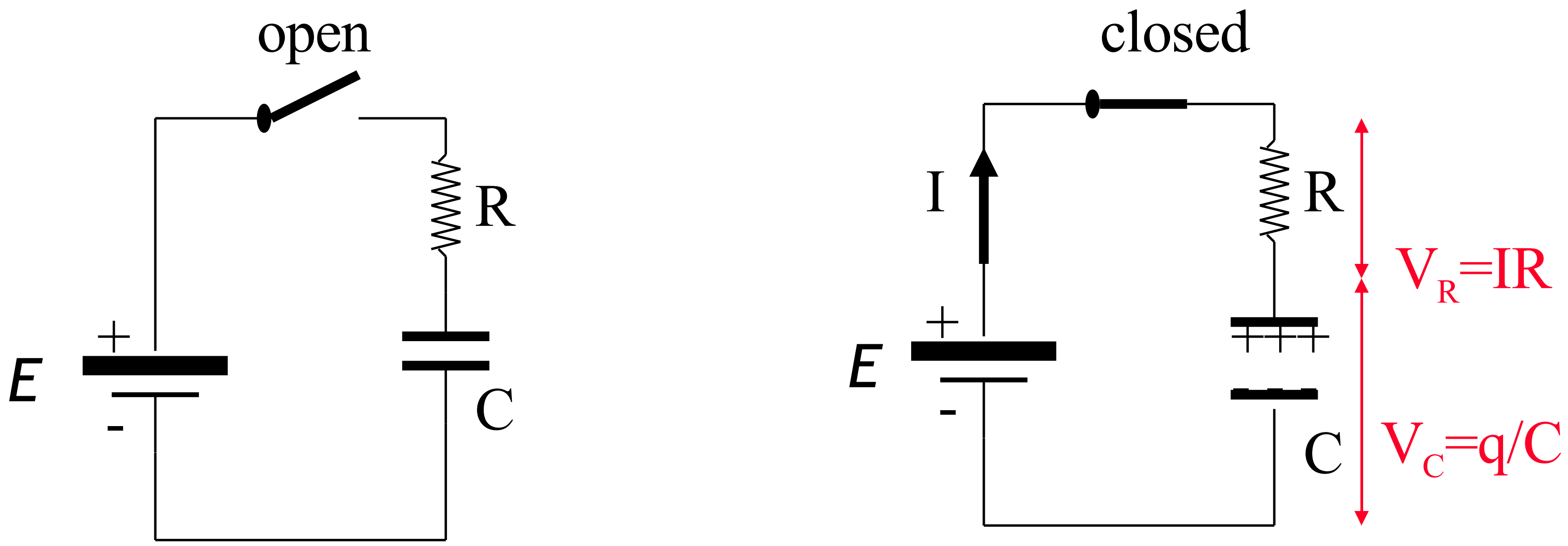
$I_3 = I_1 + I_2$

2. Sum of all ΔV's across all circuit elements in a loop must be zero.



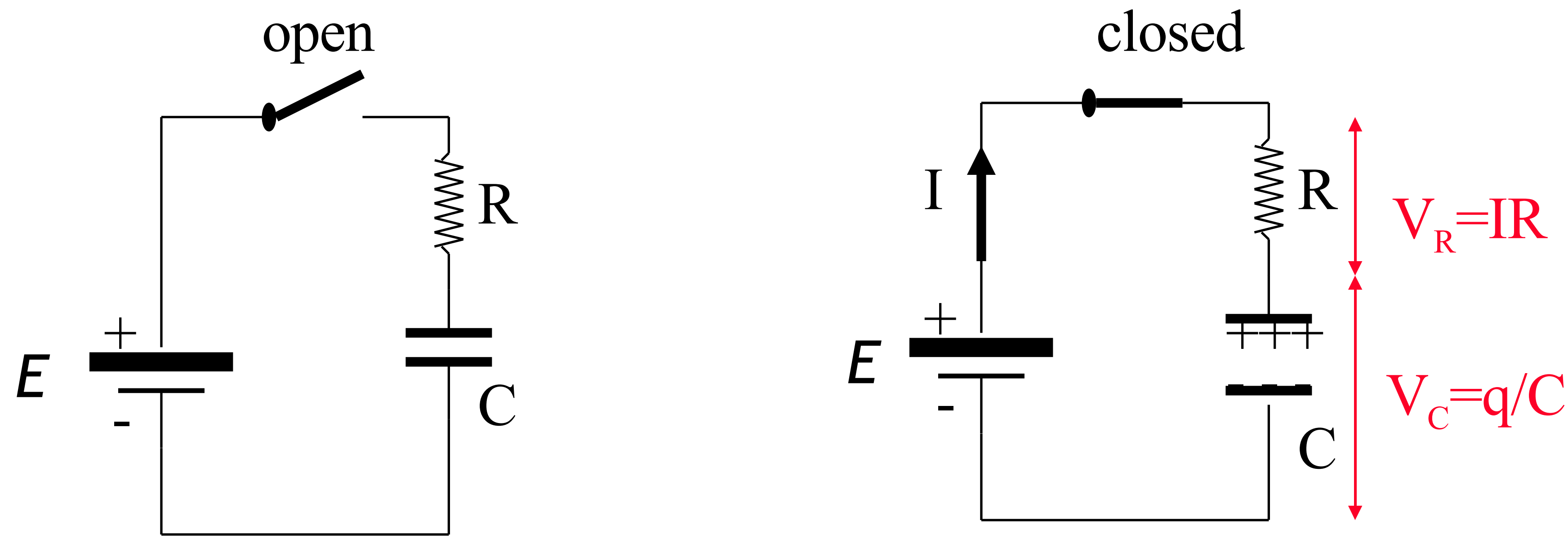
$E - Ir - IR = 0$

RC Circuits: Charging



When the switch closes, at first a high current flows: V_R is big and V_C is small.

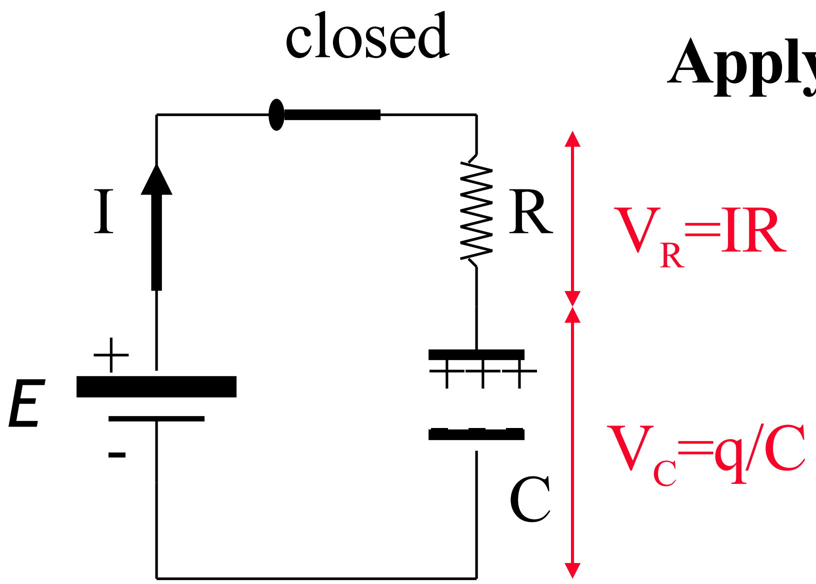
RC Circuits: Charging



$$E = IR + Q/C$$

When the switch closes, at first a high current flows:
V_R is big and V_C is small.
As q is stored in C, V_C increases.
This fights against the battery so I decreases.

RC Circuits: Charging



Apply the loop law: $\mathcal{E}$ – IR - q/C = 0

Take the derivative of this with respect to time:

$$-R\frac{dI}{dt} - \frac{1}{C}\frac{dq}{dt} = 0$$

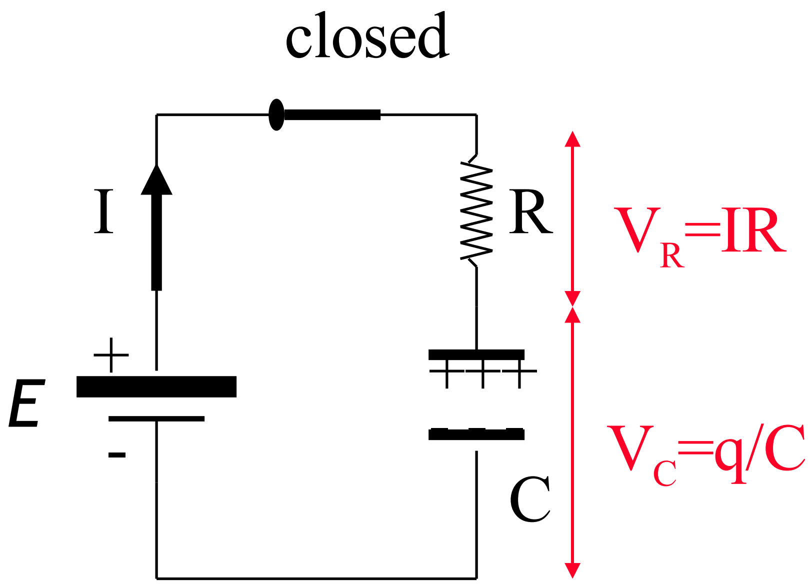
Now use dq/dt = I and rearrange:

$$\frac{dI}{dt} = -\frac{I}{RC}$$

This is a differential equation for I(t).

It is solved subject to the initial condition I(0) = $\mathcal{E}$ / R.

RC Circuits: Charging



$$\frac{dI}{dt} = -\frac{I}{RC} \Rightarrow \frac{dI}{I} = -\frac{dt}{RC}$$

$$\int dI / I = - \int \mathrm{dt} / (RC)$$

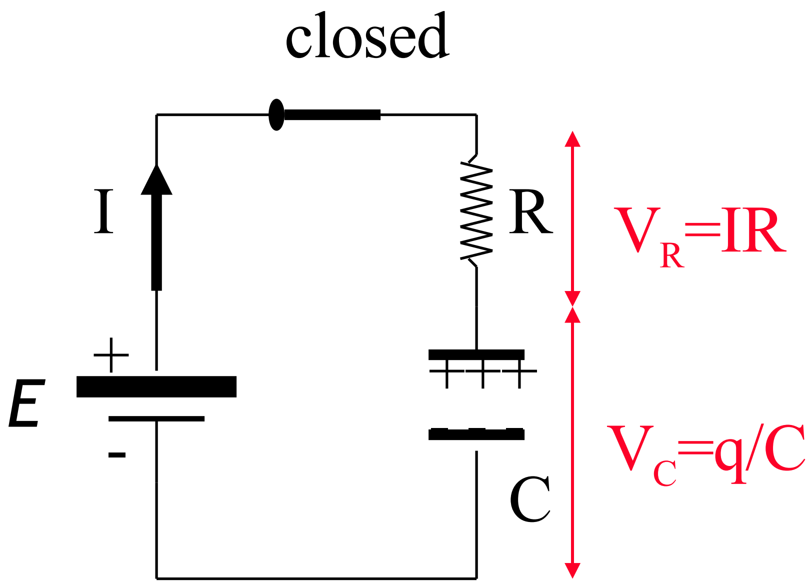
$$\ln I - \ln I(0) = - t / (RC)$$

$$I = I(0)\exp(-t / RC)$$

And $I(0) = E / R \Rightarrow$

$$I = \frac{E}{R}\exp(-t / RC)$$

RC Circuits: Charging



$$I = \frac{E}{R}\exp(-t/RC)$$

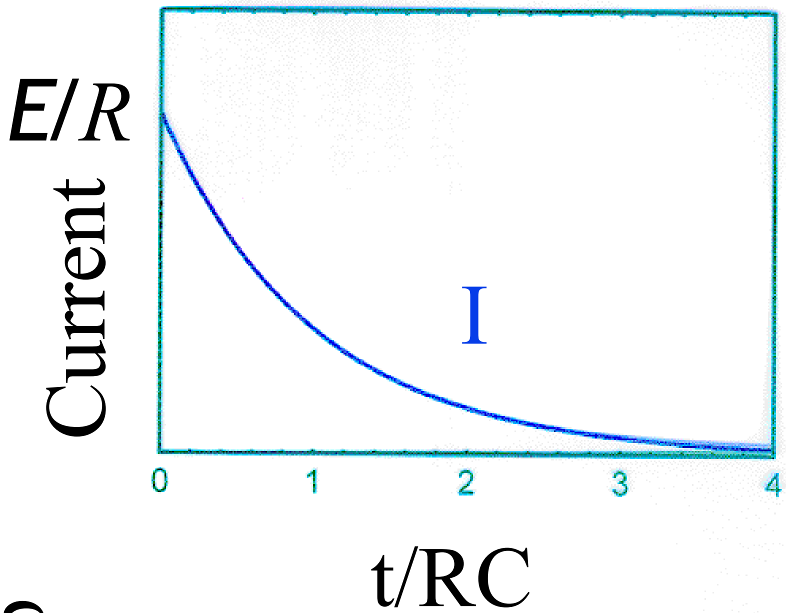
From this we get:

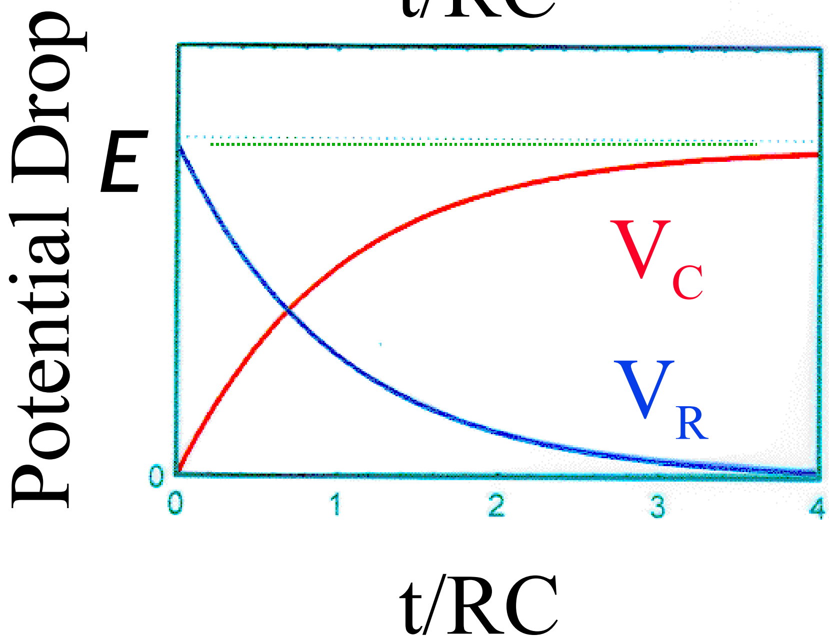
$$V_R = IR = E\exp(-t/RC)$$

$$V_C = E - V_R = E(1 - \exp(-t/RC))$$

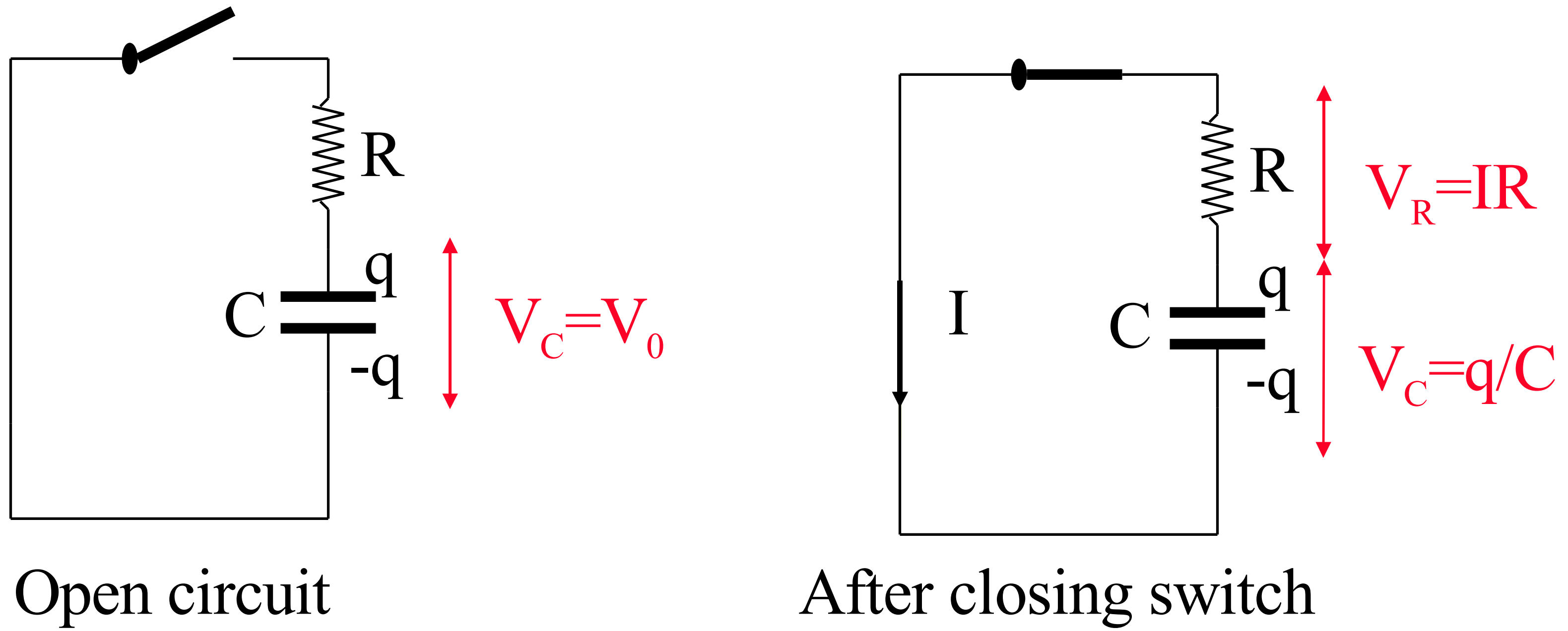
$$q = V_C C = E C\,(1 - e^{-t/RC})$$

Charging

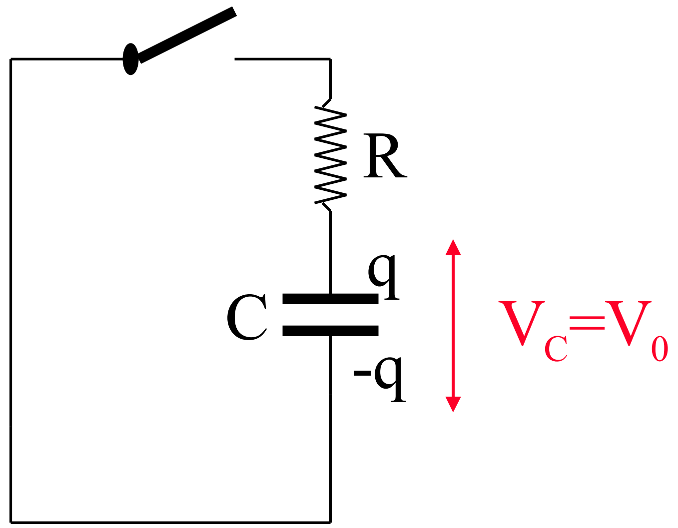




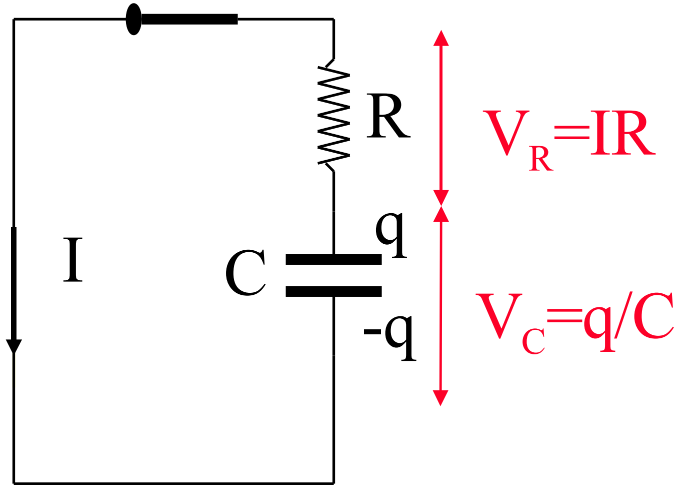
Discharging an RC Circuit



Discharging an RC Circuit



Open circuit

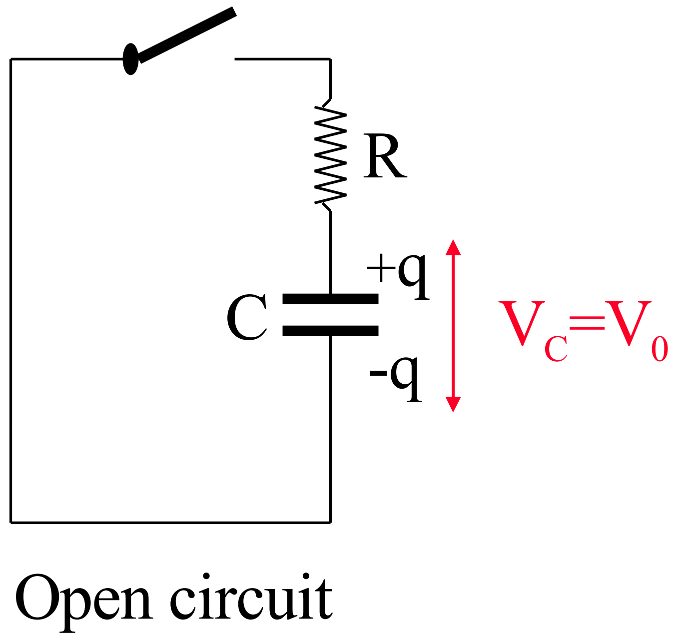


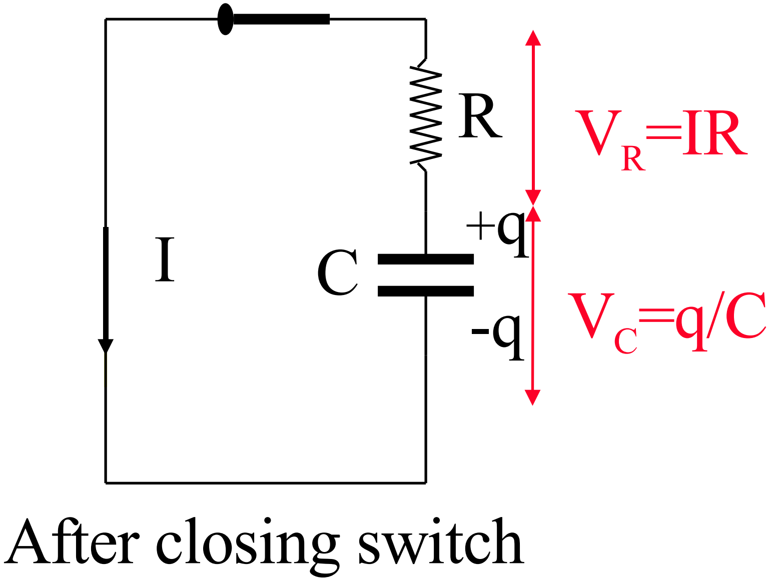
After closing switch

Current will flow through the resistor for a while.

Power $P = IV = I^2R$ will be dissipated in the resistor (as heat) while the current flows.

Discharging an RC Circuit

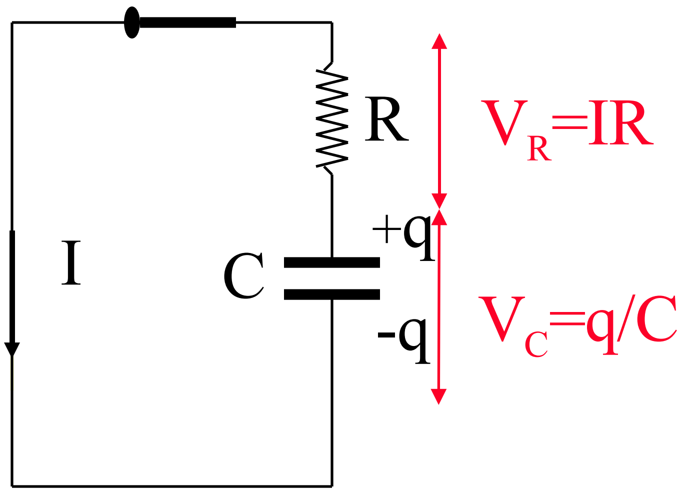




Current will flow through the resistor for a while. Eventually, the capacitor will lose all its charge, and the current will go to zero. During this transient:

$$q / C - I R = 0$$

Discharging an RC Circuit



Loop equation:

$$q/C - IR = 0 \Rightarrow I = q/(RC)$$

Take d/dt $\Rightarrow \quad \frac{dI}{dt} = -\frac{I}{RC}$

[Note that I = - dq/dt]

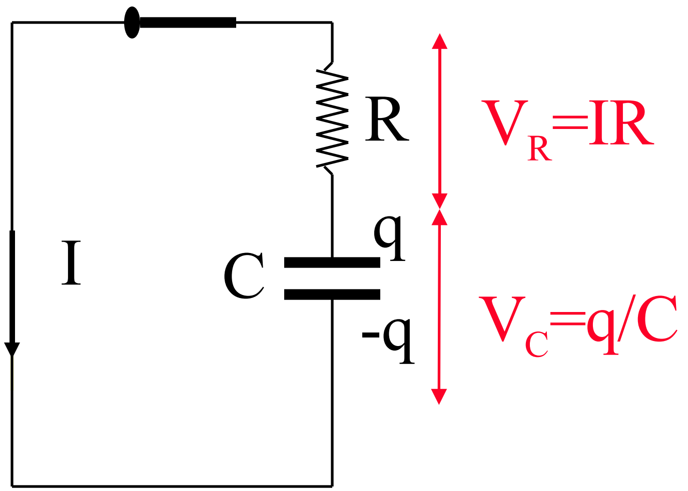
Here the current at t=0 is given by the initial voltage on the capacitor:

$$I(0) = V_0/R = q_0/RC$$

This equation is solved very much like the other (charging case):

$$I = \frac{V_0}{R}\exp(-t/RC)$$

Discharging an RC Circuit



$$I = \frac{V_0}{R}\exp(-t/RC)$$

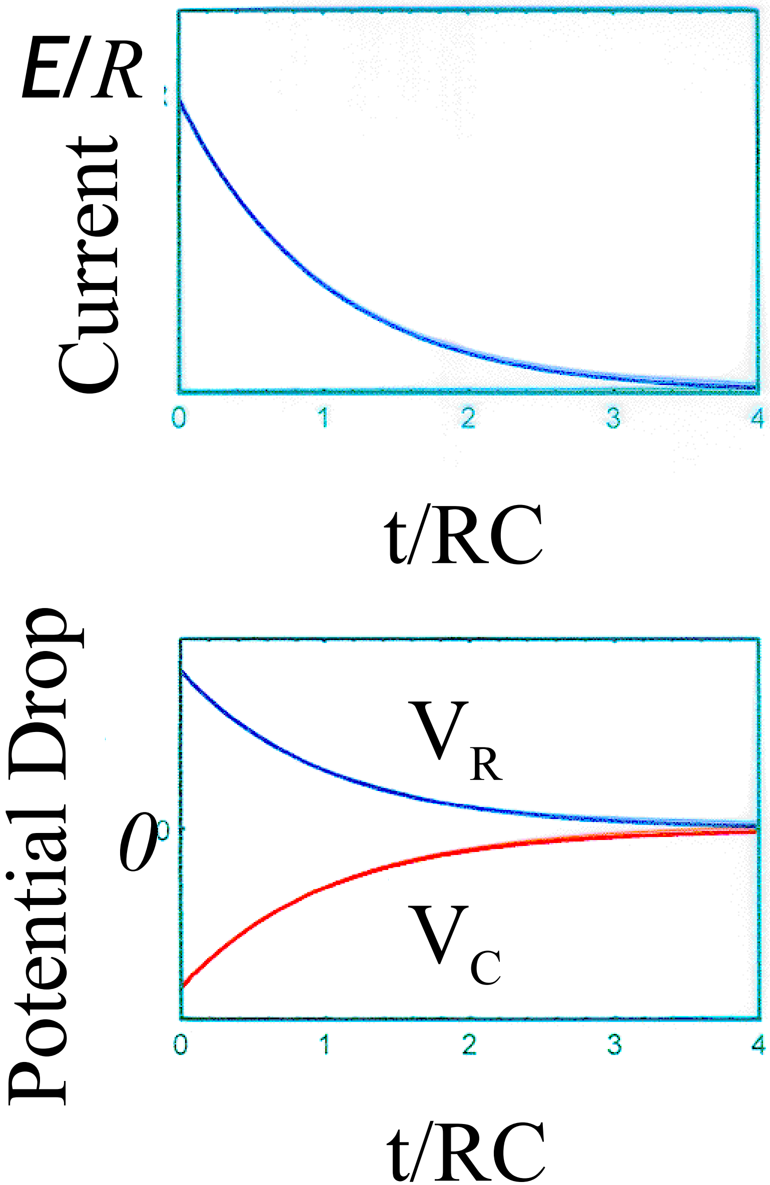
The charge on the capacitor is given by:

q/C - IR = 0 so q = C IR [q = C V]

$$q = CV_0 \exp(-t/RC)$$

$$= q_0 \exp(-t/RC)$$

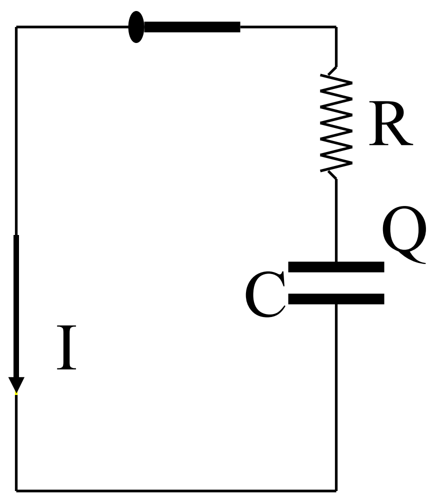
Discharging



Example: A capacitor C discharges through a resistor R.
(a) When does its charge fall to half its initial value ?

Charge on a capacitor varies as

$$Q = Q_0 \exp(-t/RC)$$

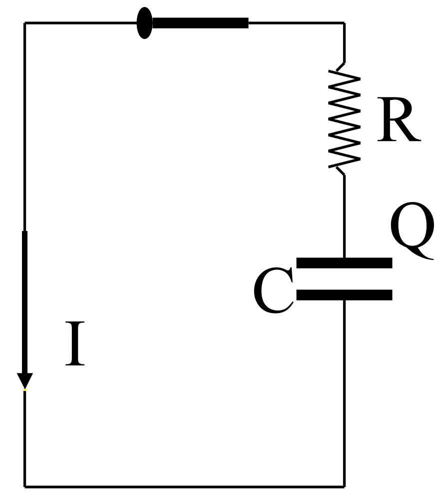


Example: A capacitor C discharges through a resistor R.
(a) When does its charge fall to half its initial value ?

Charge on a capacitor varies as

$$Q = Q_0 \exp(-t/RC)$$

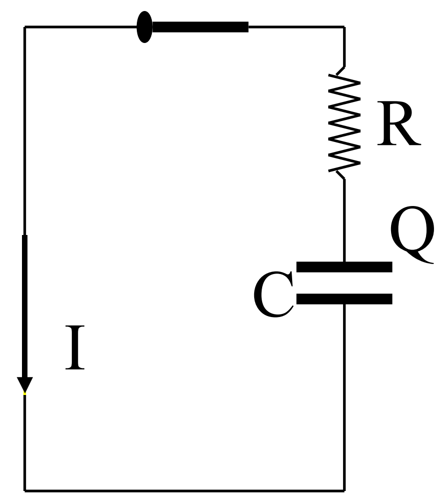
Find the time for which $Q=Q_0/2$



Example: A capacitor C discharges through a resistor R. (a) When does its charge fall to half its initial value ?

Charge on a capacitor varies as

$$Q = Q_0 \exp(-t/RC)$$



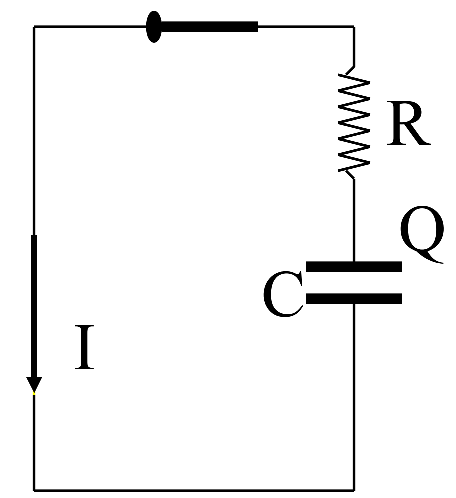
Find the time for which $Q = Q_0/2$

$$\frac{1}{2} Q_0 = Q_0 \exp(-t/RC)$$

Example: A capacitor C discharges through a resistor R.
(a) When does its charge fall to half its initial value ?

Charge on a capacitor varies as

$$Q = Q_0 \exp(-t/RC)$$



Find the time for which $Q=Q_0/2$

$$\frac{1}{2}Q_0 = Q_0 \exp(-t/RC)$$

$$\therefore\ -\ln 2 = -\frac{t}{RC}$$

$$t = (\ln 2)RC = 0.69\,RC$$

RC is the "time constant"

Example: A capacitor C discharges through a resistor R.
(b) When does the energy drop to half its initial value?

The energy stored in a capacitor is

$$U(t) = \frac{Q^2}{2C} = \frac{Q_0^2}{2C}\exp(-2t/RC) = U_0 \exp(-2t/RC)$$

Example: A capacitor C discharges through a resistor R.
(b) When does the energy drop to half its initial value?

The energy stored in a capacitor is

$$U(t) = \frac{Q^2}{2C} = \frac{Q_0^2}{2C}\exp(-2t/RC) = U_0\exp(-2t/RC)$$

We seek the time for U to drop to $U_0/2$:

$$\frac{1}{2}U_0 = U_0\exp(-2t/RC)$$

Example: A capacitor C discharges through a resistor R.
(b) When does the energy drop to half its initial value?

The energy stored in a capacitor is

$$U(t) = \frac{Q^2}{2C} = \frac{Q_0^2}{2C}\exp(-2t/RC) = U_0 \exp(-2t/RC)$$

We seek the time for U to drop to $U_0/2$:

$$\frac{1}{2}U_0 = U_0 \exp(-2t/RC)$$

$$\therefore\ t = -\ \ln 2 = RC\frac{\ln 2}{2} = 0.35RC$$