Electric Current

Chapter 27

Electric Current

Current Density

Resistivity – Conductivity - Resistance

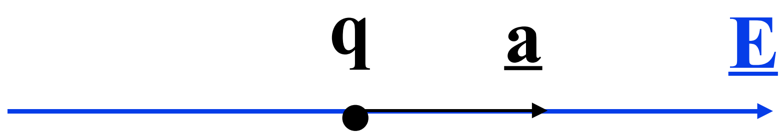
Ohm’s Law (microscopic and macroscopic)

Power Dissipated

Motion of a Point Charge in an Electric Field

A particle of mass **m** and charge **q**, placed in an electric field $\underline{\mathbf{E}}$, will experience a force $\underline{\mathbf{F}} = \mathbf{q}\,\underline{\mathbf{E}}$

$$\boxed{\underline{\mathbf{F}} = \mathbf{q}\,\underline{\mathbf{E}} = \mathbf{m}\,\underline{\mathbf{a}}}$$



The particle will accelerate with acceleration: $\underline{\mathbf{a}} = (\mathbf{q}/\mathbf{m})\,\underline{\mathbf{E}}$

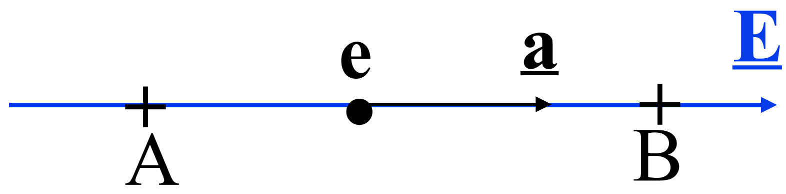
In one dimension the motion of the particle is described by:

$$x = x_0 + v_0\,t + \tfrac{1}{2}\,a\,t^2$$

$$v = v_0 + a\,t$$

$$v^2 = v_0^2 + 2\,a\,(x - x_0)$$

Motion of a Point Charge in an Electric Field



If the potential difference between A and B is 1 V, then a particle of charge e, released at A, will gain a kinetic energy of 1 electron volt [1eV] when it reaches B

$K = \frac{1}{2} m v^2$ but $v^2 = 2 a AB$ then $K = m a AB$

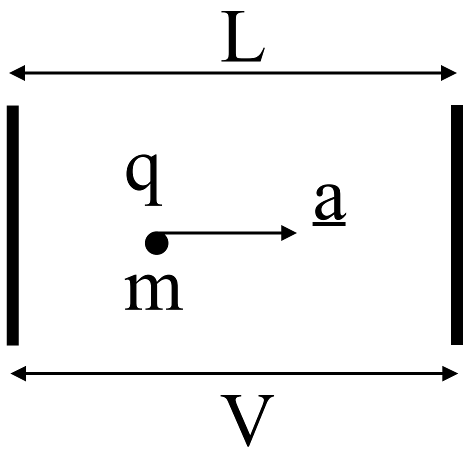
$a = (e/m) E$ and $E = V_{AB}/AB$ then $a = (e/m) V_{AB} / AB$

$K = e V_{AB}$ --- if $V_{AB} = 1$ V $\Rightarrow K = 1$ eV (electron volt)

$1 \text{ eV} = 1.6 \times 10^{-19}$ J ELECTRON VOLT

Movement of Charge Carriers

I: Between the plates of a parallel plate capacitor (vacuum)



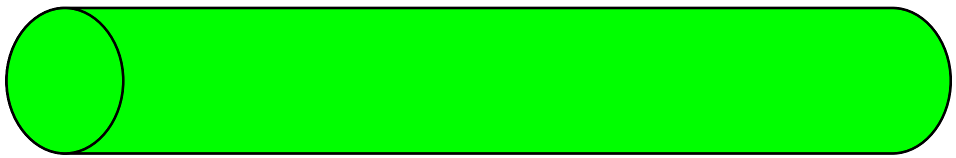
The charge accelerates with
$a = (q/m)\ E = (q/m)\ (V/L)$

If the particle (with charge q) starts at rest,
and the potential difference between the plates is V,
then the kinetic energy upon reaching the second plate will be:

$$K = qV \text{ (in electron volts)} = \tfrac{1}{2}\, m\, v^2$$

Movement of Charge Carriers

II: Inside a conductor



Inside a conductor negative electrons are the charge carriers

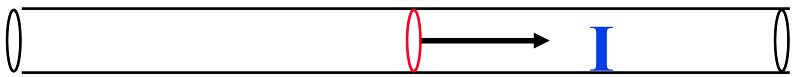
If an electric field is present, the electrons will start moving (in a direction opposite to the field).
However, the motion of the electrons will be disrupted by frequent collisions with the ions.
The net result is that the electrons acquire a slow average speed.

Current Flow in a Conductor

- Conductors are made of materials (usually metals) in which some of the electrons are free to move (not bound to the ions).

  These are called conduction electrons.
- In normal state these free electrons have random, Brownian motion in the material.
- Electrons move under the influence of an E field.
- The individual motion of electrons is still quasi-random. However, a net average flow of charge is set up when an E field is applied.

Electric current



a wire

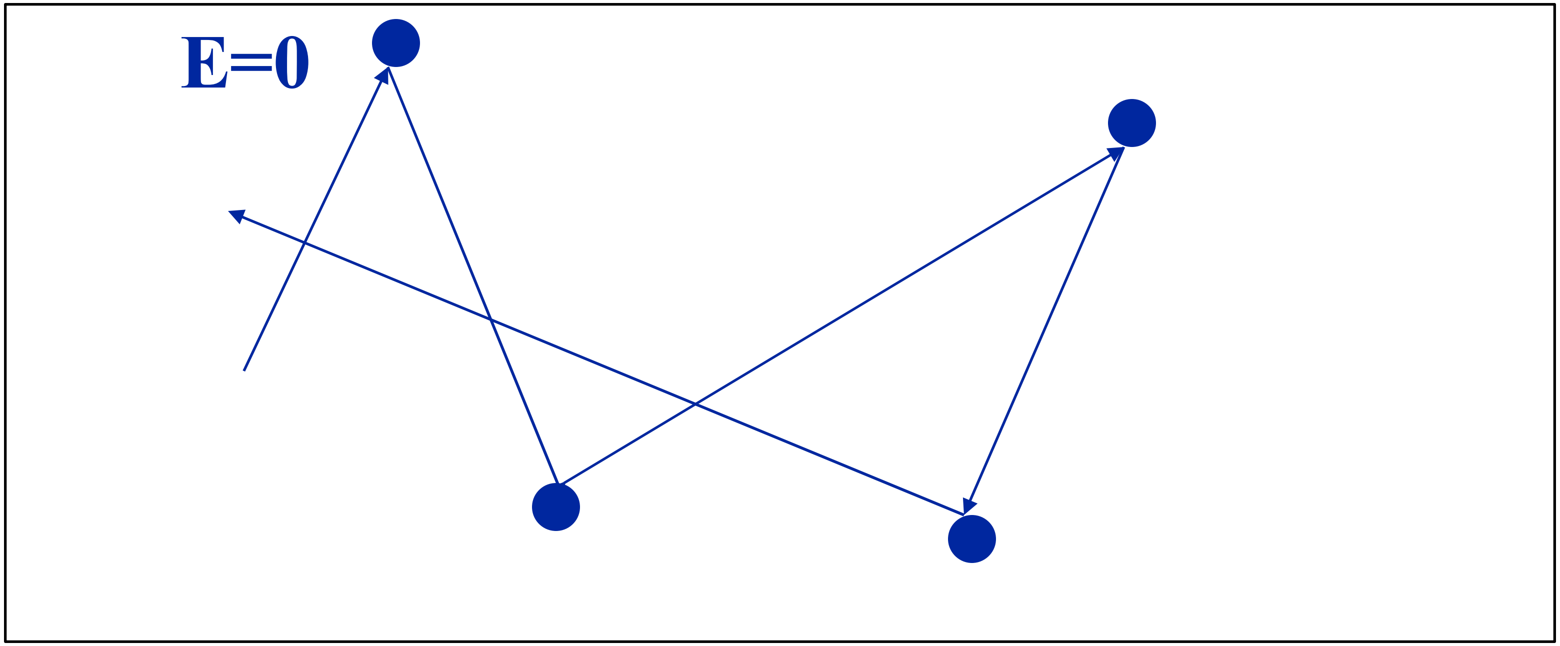
dq passes through in time dt

- We define the electric current as the movement of charge, across a given area, per unit time:

$$I = dq / dt$$

- SI unit of current: 1 C/s = 1 **Ampere** (Amp)
- The direction of the current is the direction in which positive charges would move.
- Electrons move opposite to the direction of the current.

Influence of electric field on flow of electrons



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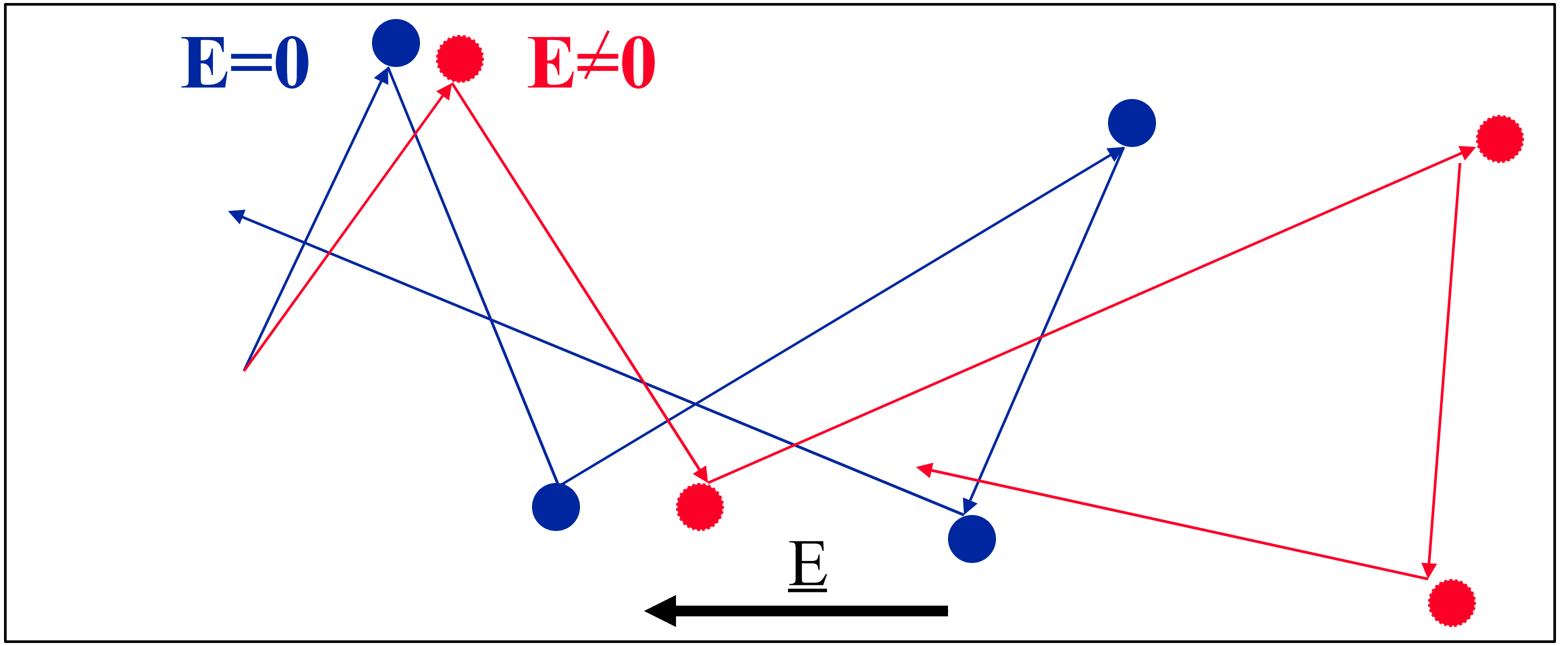
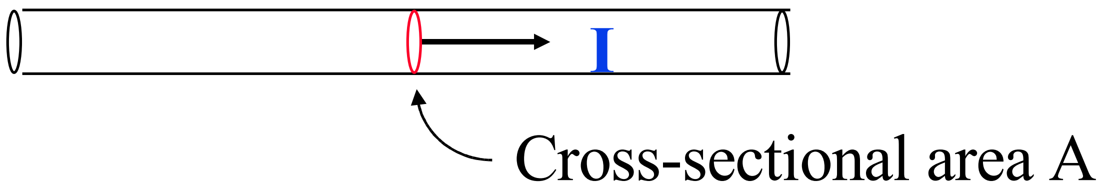


- An electric field modifies the trajectories of electrons between collisions.
- When E is nonzero, the electrons move almost randomly after each bounce, but gradually they drift in the direction opposite to the electric field.

Current density



- If current **I** flows through a surface A, the current density **J** is defined as the current per unit area:

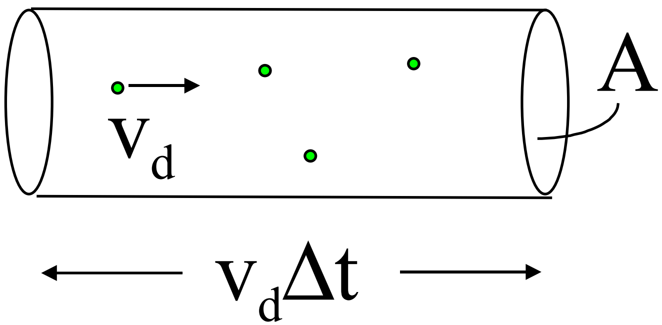
$$\mathbf{J} = \mathbf{I} / \mathbf{A}$$

- After an electron collides with an ion, it will accelerate under an E field with $a = e E / m$.
- Suppose the average time between collisions is τ.

  Then the average velocity is $v_d = a \tau = e E \tau / m$.

  This velocity is called the electron drift velocity

Current density



Density of electrons: n

Number of electrons: $N = n(A v_d \Delta t)$

- Construct the above volume.
- In time Δt all the electrons in it move out through the right end.
- Hence the charge per unit time (the current) is

$$I = (N\, e) / \Delta t = n\, e\, A\, v_d\, \Delta t / \Delta t = n\, e\, A\, v_d$$

- The current density is

$$J = I / A = n\, e\, v_d = (n\, e^2 \tau / m)\, E$$

Example: What is the drift velocity of electrons in a Cu wire 1.8 mm in diameter carrying a current of 1.3 A?

In Cu there is about one conduction electron per atom.
The density of Cu atoms is $n = 8.49x10^{28} m^{-3}$

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$$v_d = 3.8 \cdot 10^{-5}\ m/s$$

Much less than one millimeter per second!

Ohm's Law

- We found that $J = (ne^2\tau/m)E$, that is, that the current **$\underline{J}$** is proportional to the applied electric field **$\underline{E}$** *(both are vectors):*

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σ is the "Conductivity", $\sigma = J / E$.

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- It is useful to turn this around and define the

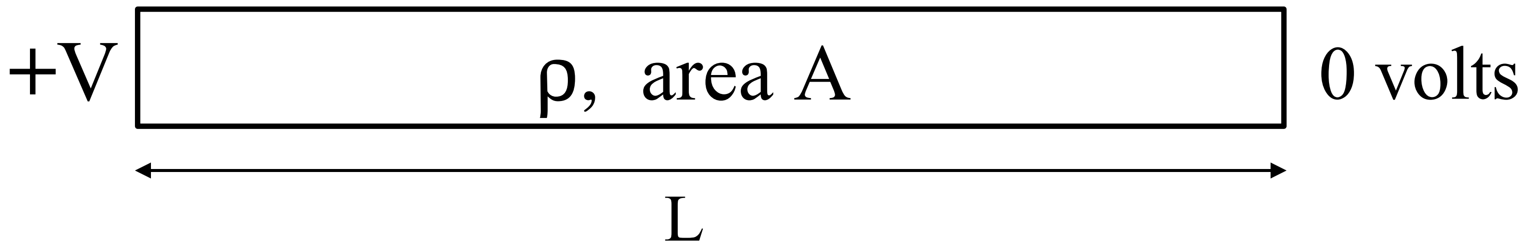
"Resistivity" as $\rho = E/J = 1/\sigma$.

Units of ρ are $(V/A)m$

Ohm's Law

σ and ρ are dependent only on the material,
- not its length or area.

However, consider a metal rod of resistivity ρ:



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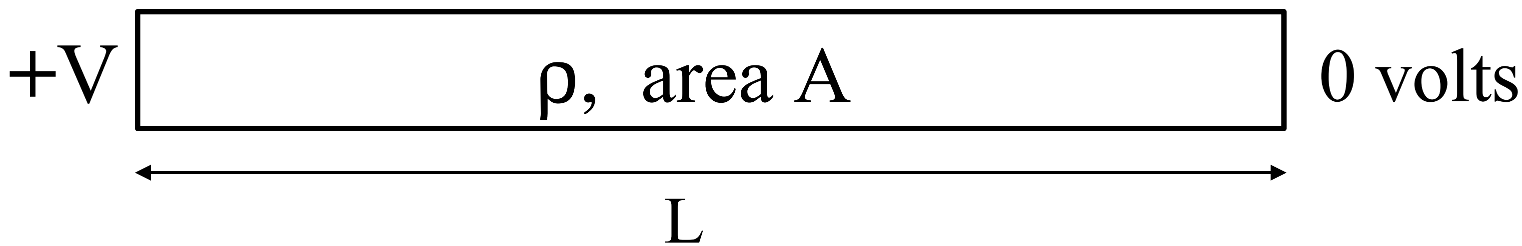

$$\mathbf{E} = \rho\, \mathbf{J}$$

$$(V/L) = \rho\, (I/A)$$

$$V = (\rho L/A)\, I$$

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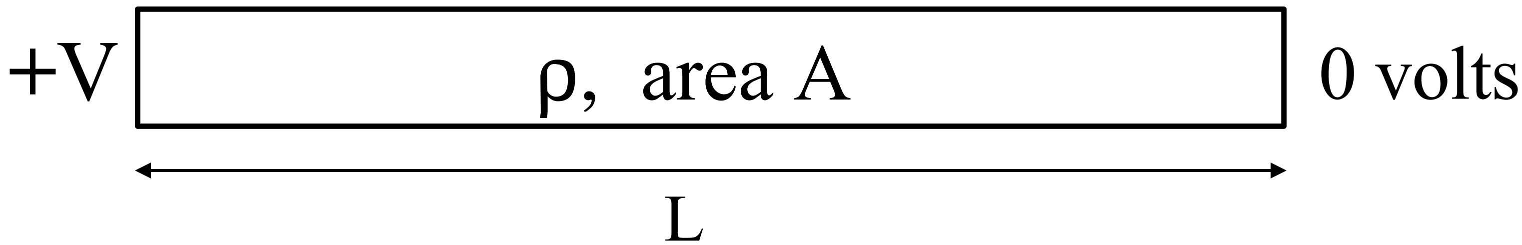

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$$V = IR$$

Ohm's Law

The macroscopic form **V = I R** is the most commonly used form of Ohm's Law.

R is the "**Resistance**"

It depends on the material type and shape:

R = ρ L / A ***Units: ohms, (Ω).***

As ρ = R A / L, common units for the resistivity ρ are ***Ohm-meters***.

Similarly, common units for the conductivity σ = 1 / ρ are ***(Ohm m)***$^{-1}$ or **Mho/m**

Ohm's Law

$\underline{J} = \sigma\, \underline{E}$ microscopic form

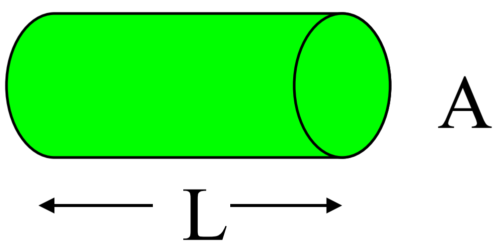
σ = conductivity **$\rho = 1/\sigma$ = resistivity**

σ and $\rho = 1/\sigma$ are dependent only on the material,

(NOT on its length or area)

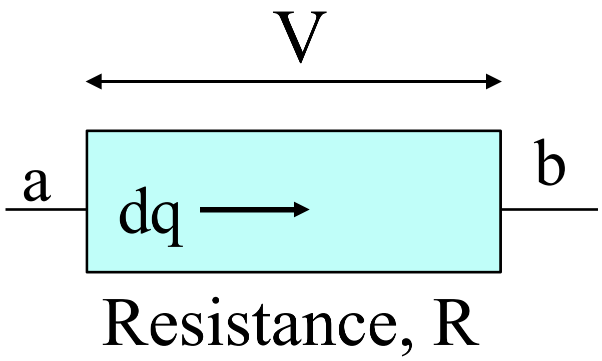
$V = I\,R$ macroscopic form

R depends on the material type and shape



$R = \rho\, L / A$ = resistance

Electrical Power Dissipation



- **In travelling from a to b , energy decrease of dq is:**

  $$dU = dq\, V$$

- **Now,** $dq = I\, dt$
- **Therefore,** $dU = I\, dt\, V$
- **Rate of energy dissipation is** $dU / dt = I\, V$
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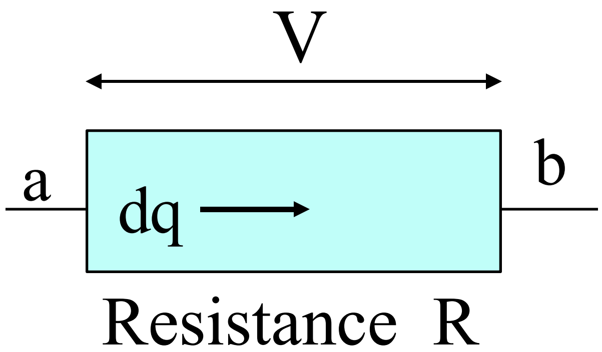

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- **Hence,**

  $$P = I\ V$$

  or

  $$P = I^2 R = V^2 / R$$

Resistivities of Selected Materials

Material	Resistivity [Ω m]
Aluminum	2.65×10^{-8}
Cooper	1.68×10^{-8}
Iron	9.71×10^{-8}
Water (pure)	2.6×10^{5}
Sea Water	0.22
Blood (human)	0.70
Silicon	640
Glass	$10^{10} – 10^{14}$
Rubber	$10^{13} – 10^{16}$

What is the resistance of a Cu wire, 1.8 mm in diameter, and 1 m long ?.

$$R = \rho\, L / A \Rightarrow R = (1.68x10^{-8})\ \ 1 / \pi\, (0.0009)^2\ \Omega$$

$$R = 6.6x10^{-3}\ \Omega$$

What is the voltage difference between the extremes of a Cu wire, 1.8 mm in diameter, and 1 m long, when the current is 1.3 A ?.

$$V = I\, R = (1.3\ A)\ 6.6x10^{-3}\ \Omega = 8.6x10^{-3}\ V$$

What is the power dissipated in a Cu wire, 1.8 mm in diameter, and 1 m long, when the current is 1.3 A ?.

$$P = I^2\, R = (1.3)^2\ 6.6x10^{-3}\ W = 1.12x10^{-2}\ W$$