Electric Potential

Chapter 25

The Electric Potential

Equipotential Surfaces

Potential due to a

Distribution of Charges

Calculating the Electric Field

From the Potential

ELECTRICAL POTENTIAL DIFFERENCE

Electrical Potential = Potential Energy per Unit Charge

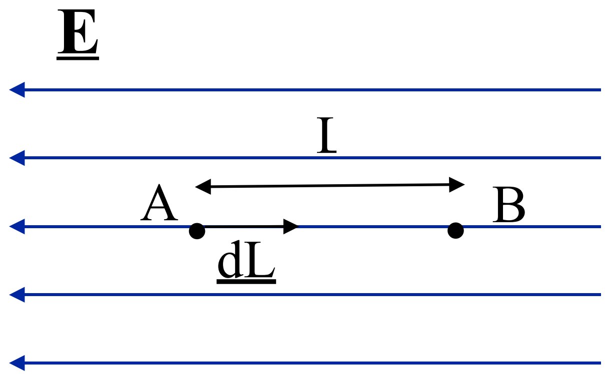
$$\Delta V_{AB} = \Delta U_{AB} / q$$

$$\Delta V_{AB} = \Delta U_{AB} / q = -(1/q)\int_A^B q\,\underline{E}\,.\,\underline{dL} = -\int_A^B \underline{E}\,.\,\underline{dL}$$

$$\Delta V_{AB} = -\int_A^B \underline{E}\,.\,\underline{dL}$$

ΔV_{AB} = Electrical potential difference between the points A and B

ELECTRICAL POTENTIAL IN A CONSTANT FIELD E



$$\Delta V_{AB} = \Delta U_{AB} / q$$

The electrical potential difference between A and B equals the work per unit charge necessary to move a charge +q from A to B

$$\Delta V_{AB} = V_B - V_A = -W_{AB}/q = -\int \underline{E}.\underline{dl}$$

But E = constant, and $\underline{E}.\underline{dl} = -1\ E\ dl$,

then:

$$\Delta V_{AB} = -\int \underline{E}.\underline{dl} = \int E\ dl = E \int dl = E\ L$$

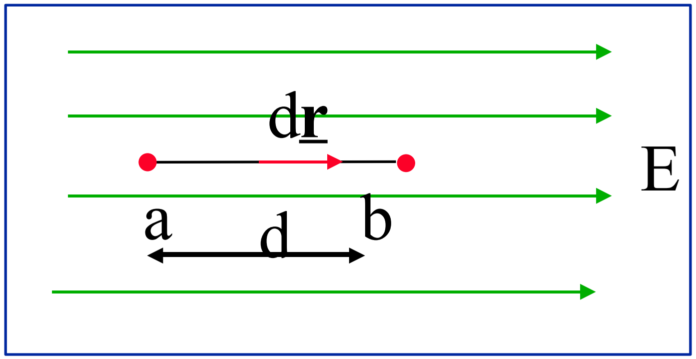
$$\Delta V_{AB} = E\ L$$

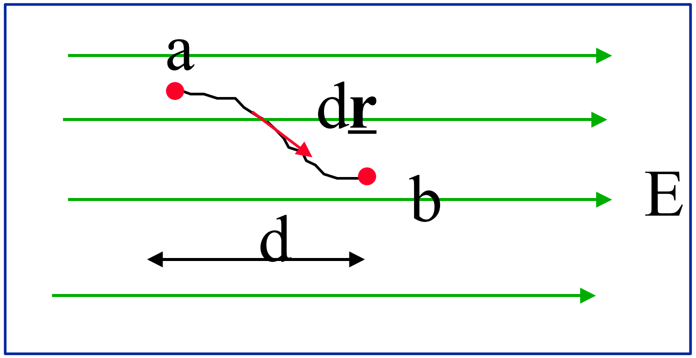
$$\Delta U_{AB} = q\ E\ L$$

Example: Electric potential of a uniform electric field

$$\Delta V = V_b - V_a = -\int_a^b \vec{E} \bullet d\vec{r}$$

$$= -\int_a^b E\,dx = -Ed$$



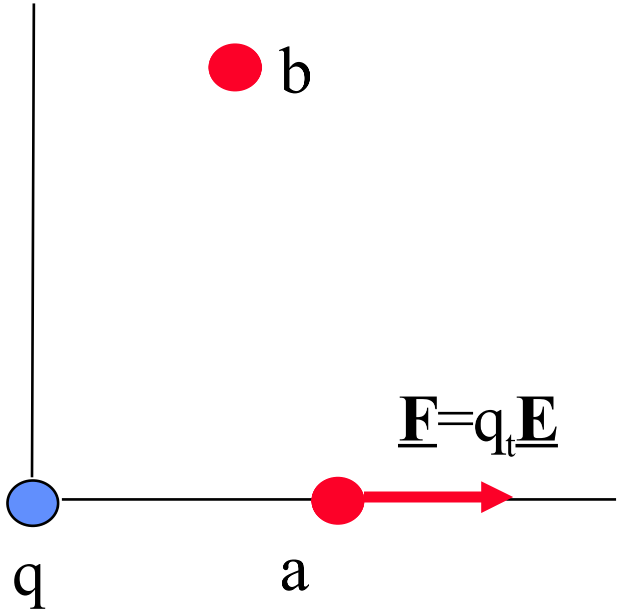


A positive charge would be pushed from regions of high potential to regions of low potential.

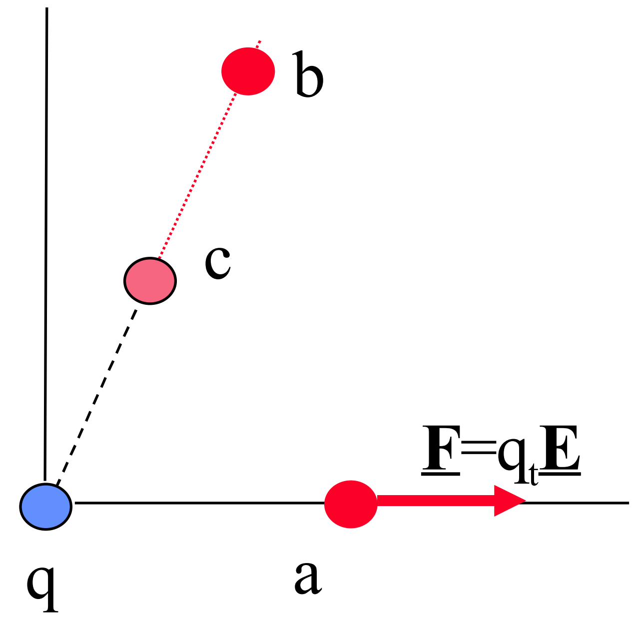
The Electric Potential

Point Charge q ≡ 

What is the electrical potential difference between two points (a and b) in the electric field produced by a point charge q.



The Electric Potential



Place the point charge q at the origin.
The electric field points radially outwards.

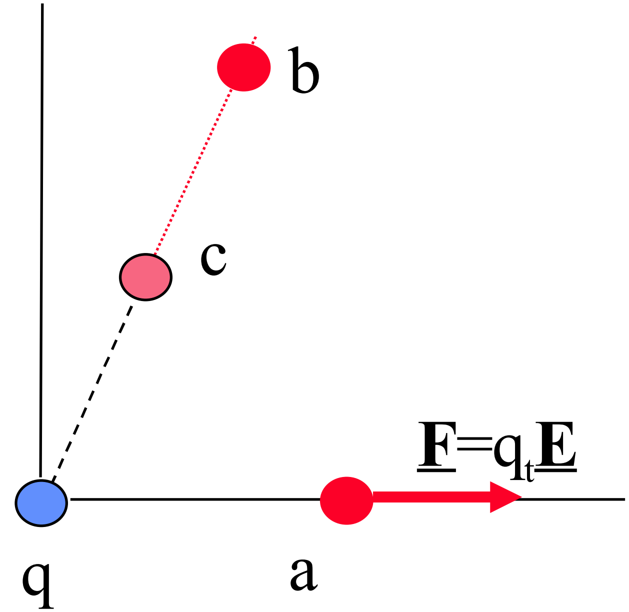
First find the work done by q's field when q_t is moved from a to b on the path a-c-b.

W = W(a to c) + W(c to b)

W(a to c) = 0 because on this path $\vec{F} \perp d\vec{r}$

$$W(c \text{ to } b) = \int_{\vec{r}_c}^{\vec{r}_b} q_t \vec{E}(\vec{r}) \bullet d\vec{r} = \int_{r_a}^{r_b} q_t E(r) dr = kq_t q \int_{r_a}^{r_b} \frac{dr}{r^2}$$

The Electric Potential



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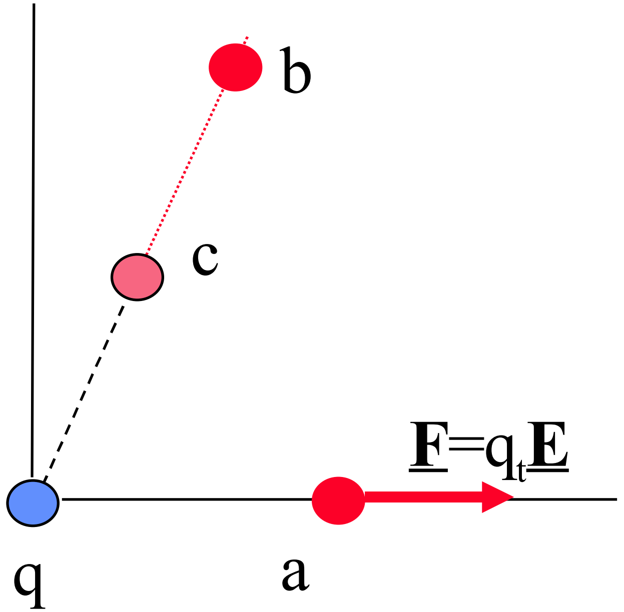
hence $\mathbf{W} = kq_t q \left(\frac{1}{r_a} - \frac{1}{r_b} \right)$

The Electric Potential

$$U(\vec{r}_b) - U(\vec{r}_a) = -W = kq_t q\left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$



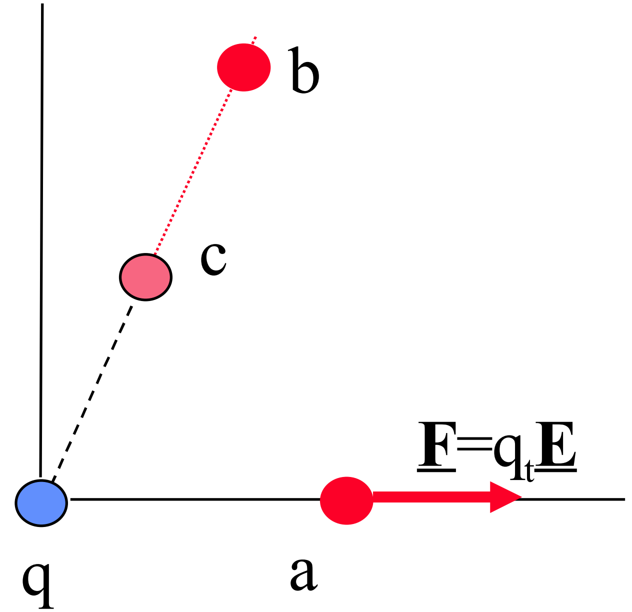
And since $\Delta V_{AB} = \Delta U_{AB} / q_t$

$$\Delta V_{AB} = k\, q\, [\, 1/r_b - 1/r_a \,]$$

The Electric Potential

$$\Delta V_{AB} = k\,q\,[\,1/r_b - 1/r_a\,]$$

From this it's natural to choose the **zero of electric potential** to be when $r_a \rightarrow \infty$



Letting **a** be the point at infinity, and dropping the subscript **b**, we get the electric potential:

$$V = k\,q\,/\,r$$

When the source charge is **q**, and the electric potential is evaluated at the point **r**.

Remember: this is the electric potential with respect to infinity

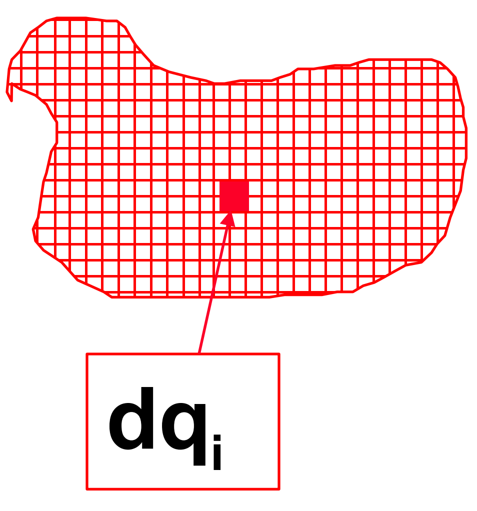
Potential Due to a Group of Charges

- For isolated point charges just add the potentials created by each charge (superposition)

- For a continuous distribution of charge …

Potential Produced by a Continuous Distribution of Charge

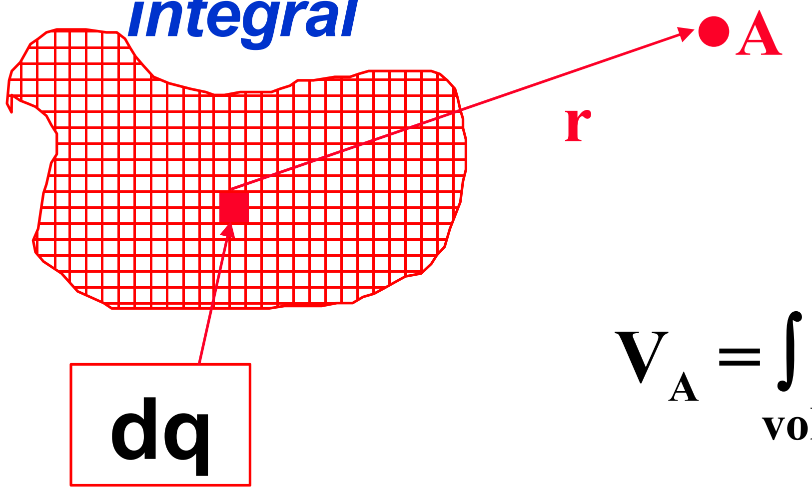
In the case of a continuous distribution of charge we first divide the distribution up into small pieces, and then we sum the contribution, to the electric potential, from each piece:



Potential Produced by a Continuous Distribution of Charge

In the case of a continuous distribution of charge we first divide the distribution up into small pieces, and then we sum the contribution, to the electric potential, from each piece:

In the limit of very small pieces, the sum is an *integral*



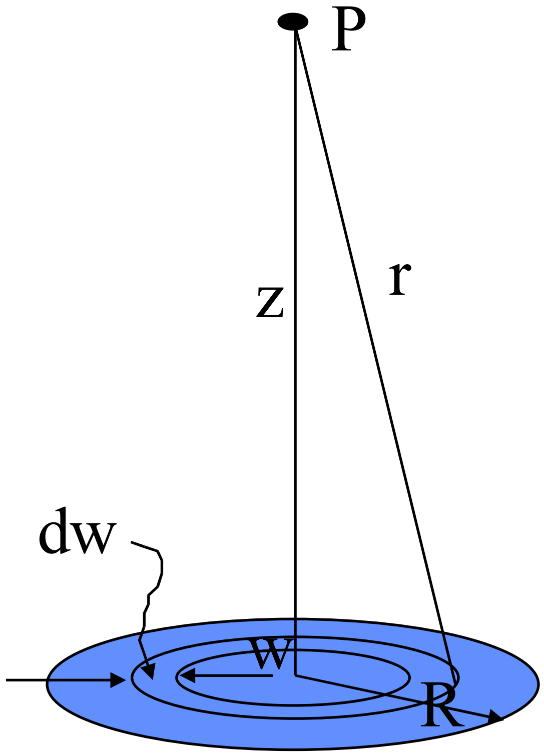
$$dV_A = k\,dq / r$$

$$V_A = \int_{vol} dV_A = \int_{vol} k\,dq / r$$

Remember $k = 1/(4\pi\varepsilon_0)$

Example: a disk of charge

Suppose the disk has radius R and a charge per unit area σ. Find the potential and electric field at a point up the z axis. Divide the object into small elements of charge and find the potential dV at P due to each bit. So here let a bit be a small ring of charge width dw and radius w.



$$dq = \sigma 2\pi w dw$$

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\frac{\sigma\, 2\pi\, w dw}{\sqrt{w^2+z^2}}$$

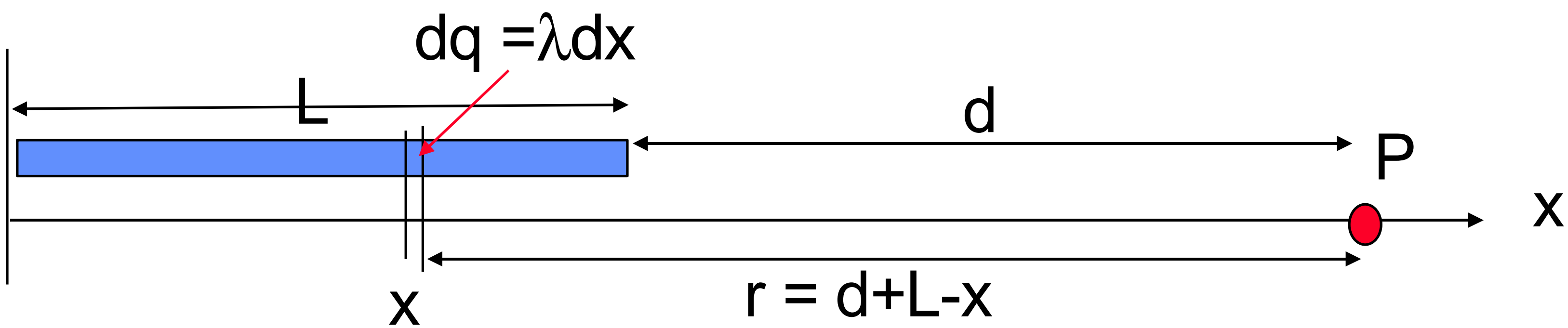
$$\therefore\ V = \int dV = \frac{\sigma}{2\varepsilon_0}\int_0^R (w^2+z^2)^{-1/2}\, w dw$$

$$V = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{R^2+z^2} - z\right)$$

Example: a line of charge

A charge density per unit length λ=400 mC/m stretches for 10 cm.

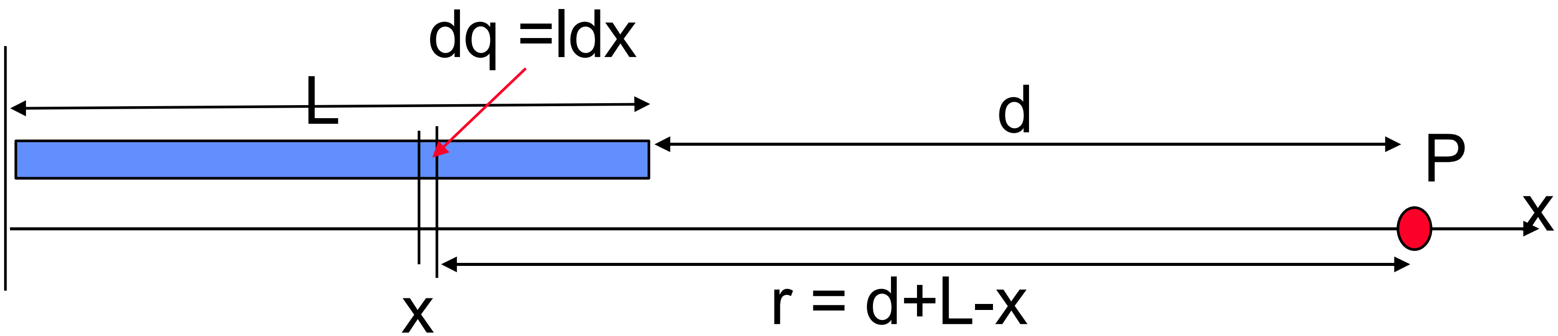
Find the electric potential at a point 15 cm from one end.



Break the charge into little bits: say a length dx at position x.

The contribution due to this bit at P is:

$$dV = \frac{k(\lambda\, dx)}{r} = \frac{k\lambda}{d + L - x}\, dx$$



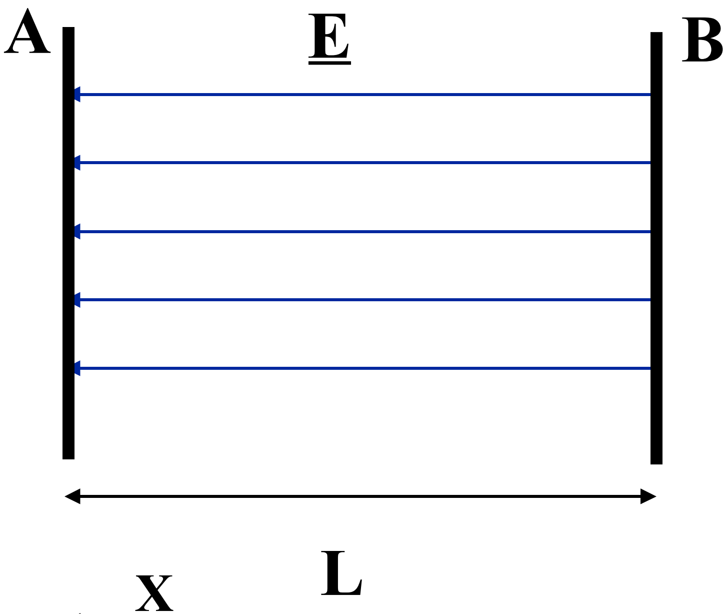
$$V = \int_{o}^{L} \frac{\lambda dx}{4\pi\varepsilon_o} \frac{1}{[(d+L)-x]}$$

$$= \{-\ln[(d+L)-x]\}_o^L = \frac{\lambda}{4\pi\varepsilon_o}\ln(\frac{d+L}{d})$$

$$= \frac{(0.44x10^{-6}C/m)}{4\pi[8.9x10^{-12}C^2/N.m^2]}\ln(\frac{0.25m}{0.15m})$$

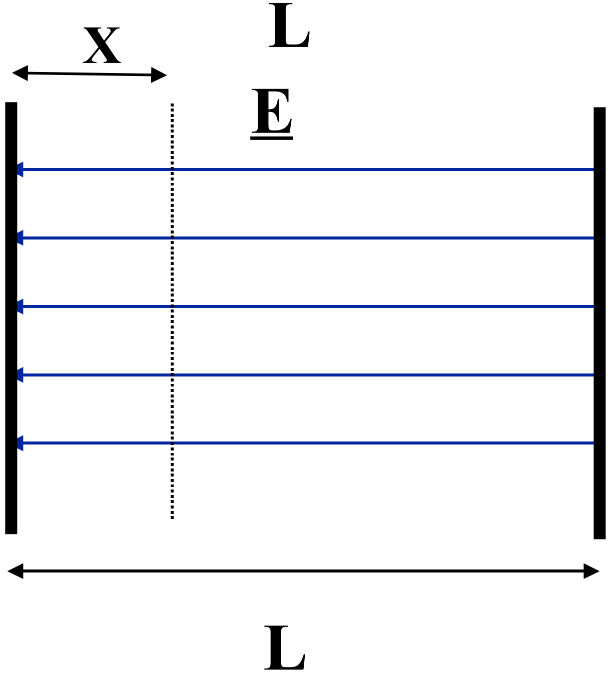
$$= 1.84x10^3 V$$

Equipotential Surfaces (lines)



Since the field E is constant

$$\Delta V_{AB} = E\,L$$



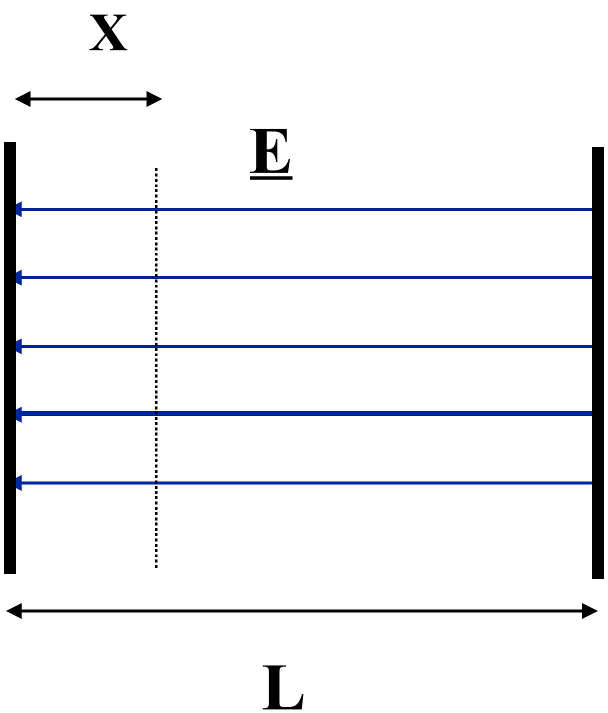
Then, at a distance X from plate A

$$\Delta V_{AX} = E\,X$$

All the points along the dashed line, at X, are at the same potential.

The dashed line is an equipotential line

Equipotential Surfaces (lines)



It takes no work to move a charge at right angles to an electric field

$$\underline{E} \perp \underline{dL} \Rightarrow \int \underline{E} \bullet \underline{dL} = 0 \Rightarrow \Delta V = 0$$

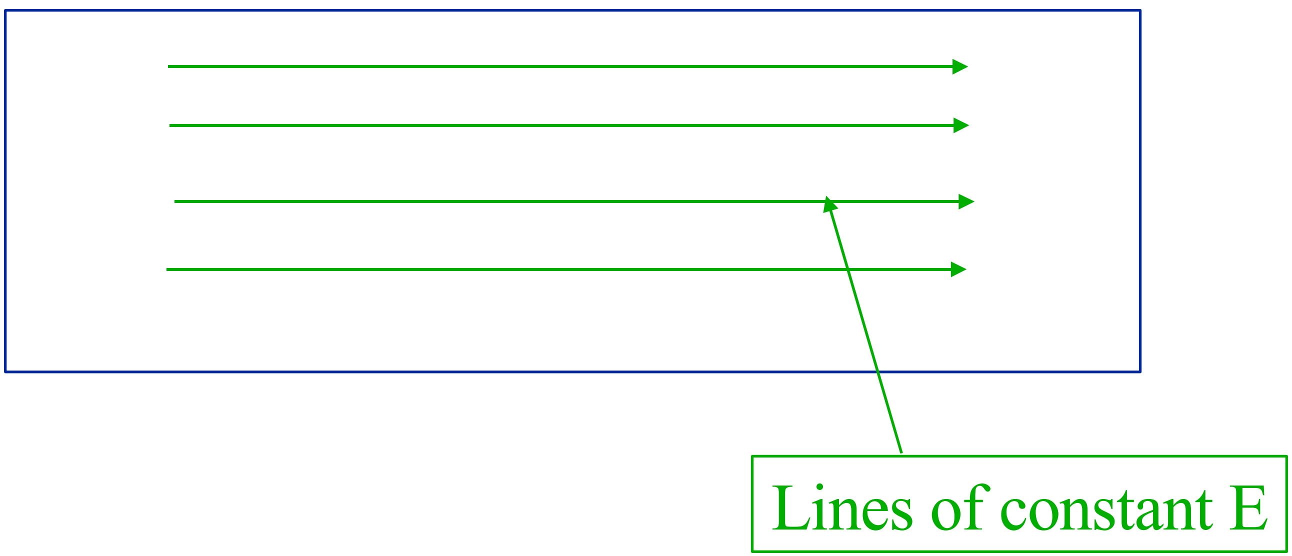
If a surface (line) is perpendicular to the electric field, all the points in the surface (line) are at the same potential. Such surface (line) is called

EQUIPOTENTIAL

EQUIPOTENTIAL ⊥ ELECTRIC FIELD

Equipotential Surfaces

We can make graphical representations of the electric potential in the same way as we have created for the electric field:



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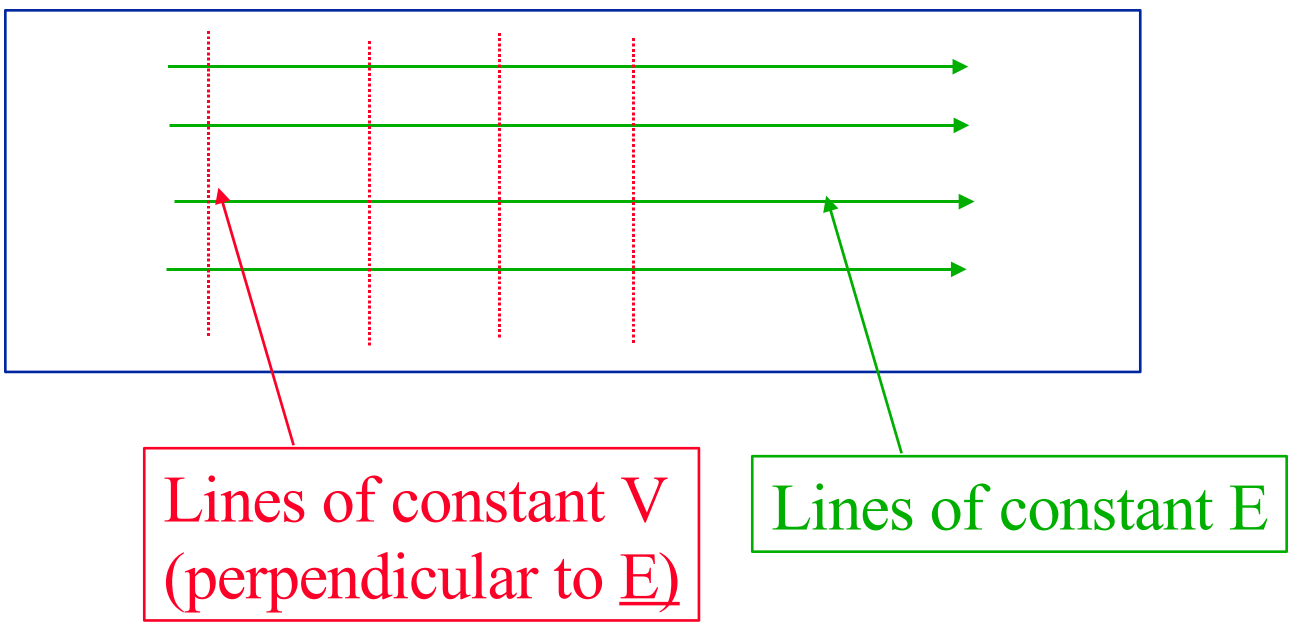

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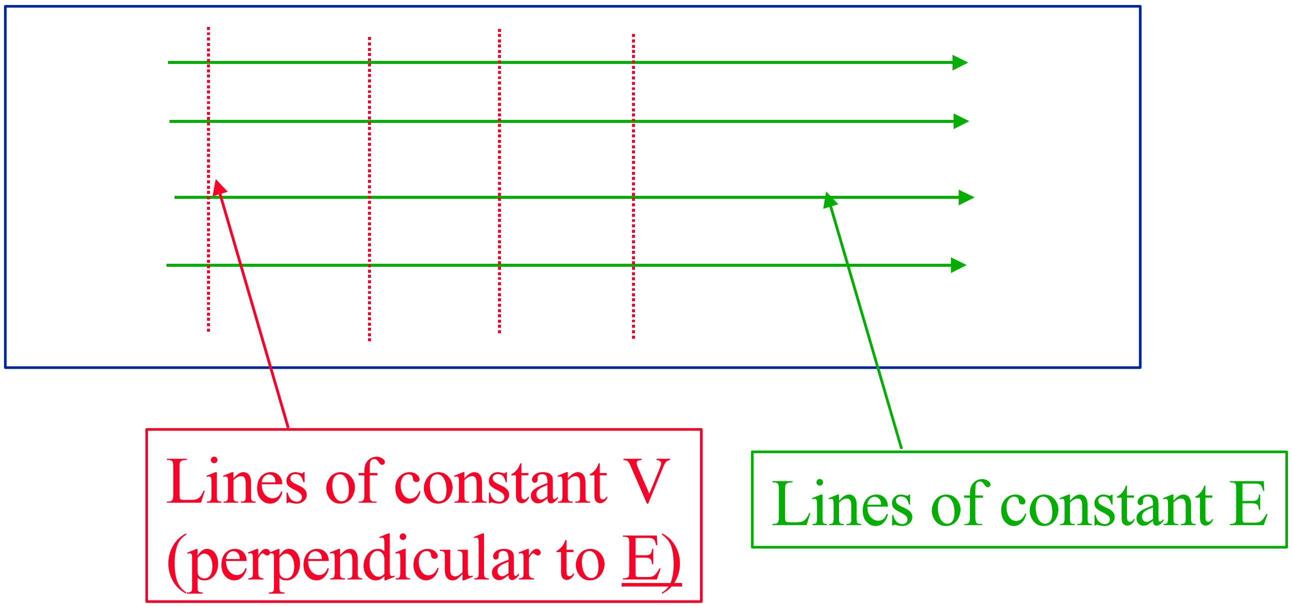
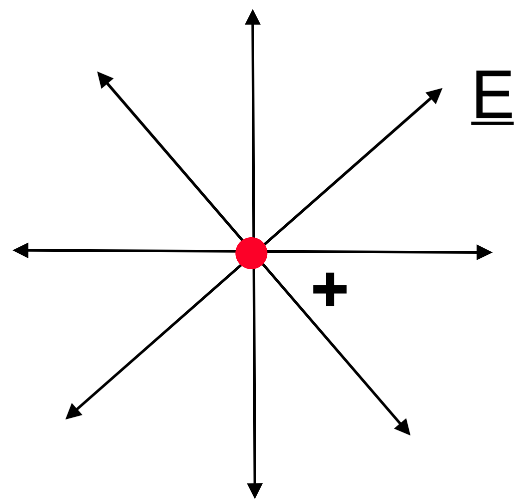


Equipotential plots are like contour maps of hills and valleys.

Equipotential Surfaces

How do the equipotential surfaces look for:

(a) A point charge?



(b) An electric dipole?

Equipotential plots are like contour maps of hills and valleys.

Force and Potential Energy

Choosing an arbitrary reference point $\underline{\mathbf{r}}_0$ (such as ∞) at which $U(\underline{\mathbf{r}}_0) = 0$, the potential energy is:

$$U(x,y,z) = -\int \vec{F} \bullet d\vec{r}$$

This can be inverted:

$$\vec{F}(x,y,z) = -(\hat{\mathbf{i}}\,\partial/\partial x + \hat{\mathbf{j}}\,\partial/\partial y + \hat{\mathbf{k}}\,\partial/\partial z)U(x,y,z)$$

The potential energy **U** is calculated from the force **F**, and conversely the force **F** can be calculated from the potential energy **U**

Field and Electric Potential

Dividing the preceding expressions by the (test) charge q we obtain:

$$V(x, y, z) = -\int \underline{F} \bullet \underline{dr}$$

$$\underline{E}(x, y, z) = -\ \underline{\nabla} V$$

$$\underline{\nabla} V = (dV/dx)\ \underline{i} + (dV/dy)\ \underline{j} + (dV/dz)\ \underline{k}$$

$$\underline{\nabla} \equiv \text{gradient}$$

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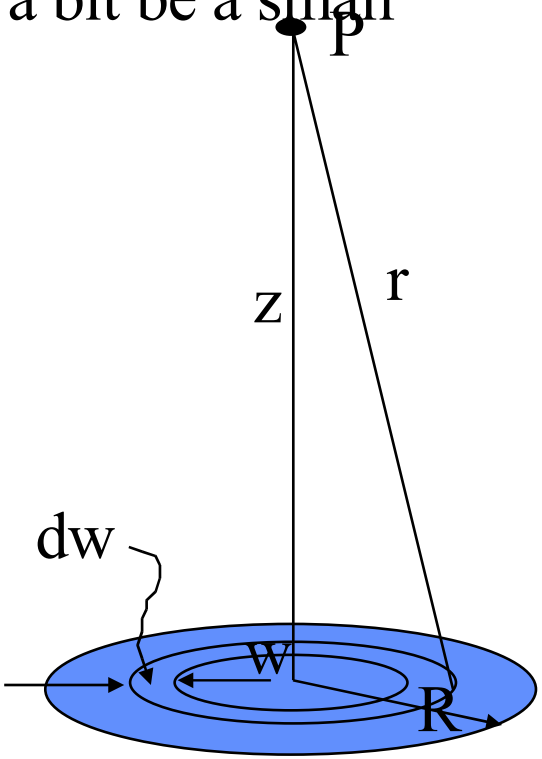


$$dq = \sigma 2\pi w dw$$

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\frac{\sigma\, 2\pi\, w dw}{\sqrt{w^2 + z^2}}$$

$$\therefore\ V = \int dV = \frac{\sigma}{2\varepsilon_0}\int_0^R (w^2 + z^2)^{-1/2}\, w dw$$

$$V = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{R^2 + z^2} - z\right)$$

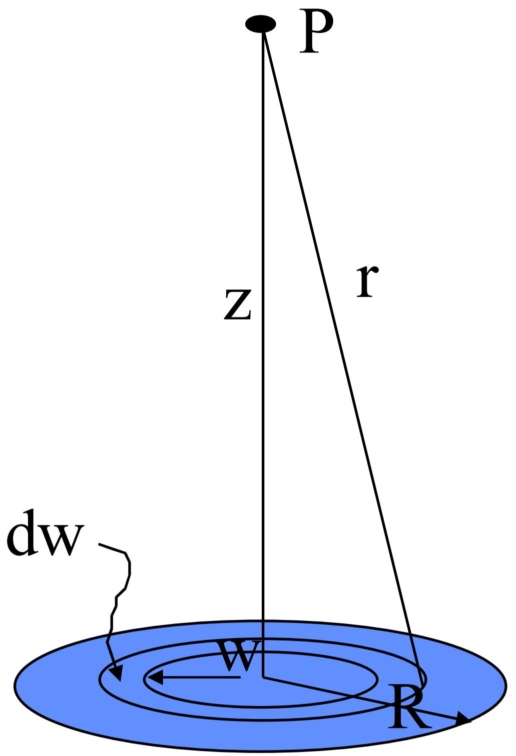
Example: a disk of charge

$$V(z) = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{R^2 + z^2} - z\right)$$

By symmetry one sees that $E_x=E_y=0$ at P.

Find E_z from

$$E_z = -\frac{\partial V}{\partial z} = -\frac{\sigma}{2\varepsilon_0}\left(\frac{z}{\sqrt{R^2 + z^2}} - 1\right)$$

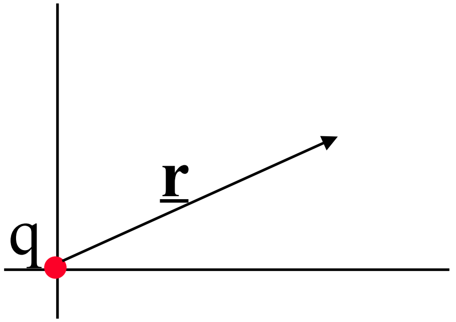


This is easier than integrating over the components of vectors. Here we integrate over a scalar and then take partial derivatives.

Example: point charge

Put a point charge q at the origin.

Find V(**r**): here this is easy: $V(\vec{\mathbf{r}}) = k\frac{q}{r}$



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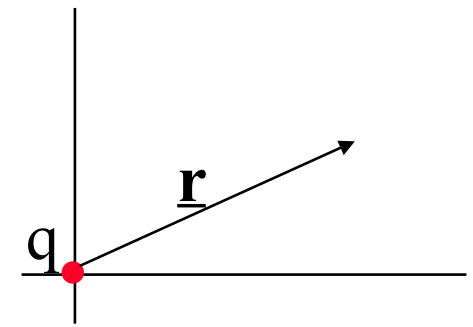

Then find **E**(**r**) from the derivatives:

$$\vec{\mathrm{E}}(\vec{\mathbf{r}}) = -(\hat{\mathrm{i}}\,\partial/\partial x + \hat{\mathrm{j}}\,\partial/\partial y + \hat{\mathrm{k}}\,\partial/\partial z)\mathrm{V}(x,y,z)$$

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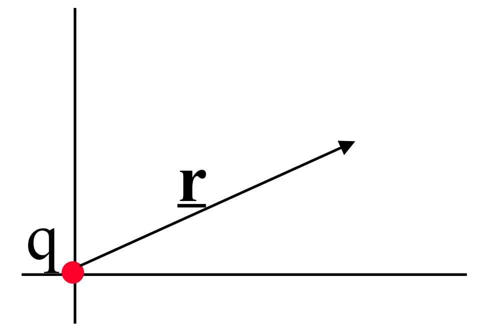

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Derivative: $$\frac{\partial}{\partial x}\frac{1}{r} = \frac{\partial}{\partial x}\frac{1}{\sqrt{x^2 + y^2 + z^2}} = -\frac{1}{2}\frac{2x}{\left(x^2 + y^2 + z^2\right)^{3/2}}$$

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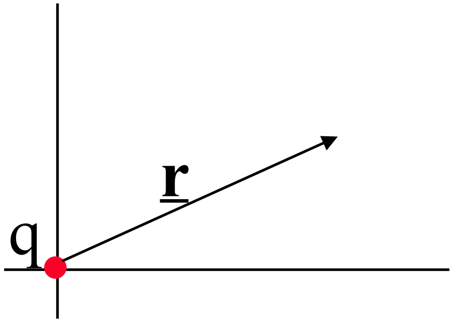

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So: $$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = kq\frac{x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}}{r^3} = kq\frac{\vec{\mathbf{r}}}{r^3} = kq\frac{\hat{\mathbf{r}}}{r^2}$$