Electric Potential

Chapter 25

Electric Potential Energy

The Electric Potential

Equipotential Surfaces

CONSERVATIVE FORCES

A conservative force "gives back" work that has been done against it

When the total work done by a force F, moving an object over a closed loop, is zero, then the force is conservative

$$\oint \underline{\mathbf{F}}.\underline{\mathbf{dr}} = 0 \iff \underline{\mathbf{F}} \text{ is conservative}$$

The circle on the integral sign indicates that the integral is taken over a closed path

The work done by a conservative force, in moving and object between two points A and B, is independent of the path taken

$\int \underline{\mathbf{F}}.\underline{\mathbf{dr}}$ **is a function of A and B only**

is NOT a function of the path selected

POTENTIAL ENERGY

Potential energy is a relative quantity, that means, it is always the difference between two values, or it is measured with respect to a reference point (usually infinity).

We will always refer to, or imply, the change in potential energy (potential energy difference) between two points.

The change ΔU_{AB} in potential energy, associated with a conservative force, is the negative of the work done by that force, as it acts (over any path) from point A to point B

$$\Delta U_{AB} = -\int_A^B \underline{\mathbf{F}}.\underline{\mathbf{dr}}$$

$\Delta U_{AB} = U_B - U_A$ = potential energy difference between A and B

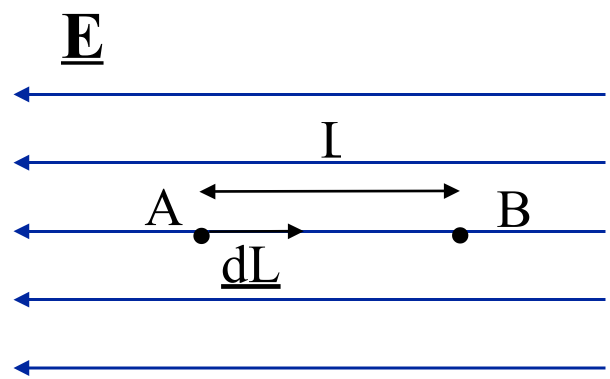
POTENTIAL ENERGY ASSOCIATED WITH THE ELECTRIC FORCE

In general: $\Delta U_{AB} = U_B - U_A = -W_{AB} = -\int \underline{F}.\underline{dl}$

In the case of the electric force $\underline{F} = q\,\underline{E}$, then:

$$\Delta U_{AB} = U_B - U_A = -W_{AB} = -\int_A^B q\,\underline{E}.\underline{dl}$$

POTENTIAL ENERGY IN A CONSTANT FIELD E



The potential energy difference between A and B equals the work necessary to move a charge +q from A to B

$$\Delta U_{AB} = U_B - U_A = -W_{AB} = -\int q\, \underline{E}.\underline{dl}$$

But E = constant, and $\underline{E}.\underline{dl}$ = -1 E dl, then:

$$\Delta U_{AB} = -\int q\, \underline{E}.\underline{dl} = \int q\, E\, dl = q\, E \int dl = q\, E\, L$$

$$\boxed{\Delta U_{AB} = q\, E\, L}$$

ELECTRICAL POTENTIAL DIFFERENCE

The potential energy ΔU depends on the charge being moved. In order to remove this dependence, we introduce the concept of electrical potential ΔV

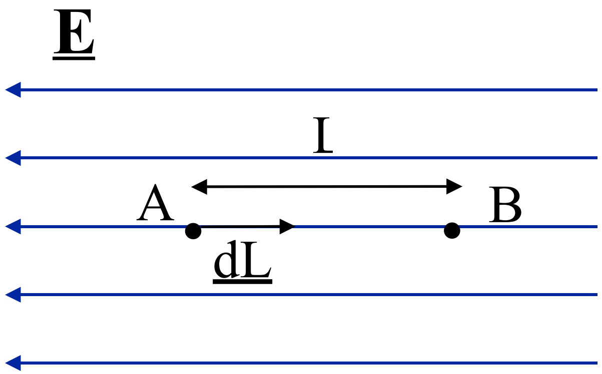
$$\boldsymbol{\Delta V_{AB} = \Delta U_{AB} / q}$$

Electrical Potential = Potential Energy per Unit Charge

$$\Delta V_{AB} = \Delta U_{AB} / q = -(1/q) \int q \, \underline{\mathbf{E}} \cdot \underline{\mathbf{dL}} = -\int_A^B \underline{\mathbf{E}} \cdot \underline{\mathbf{dL}}$$

ΔV_{AB} = Electrical potential difference between the points A and B

ELECTRICAL POTENTIAL IN A CONSTANT FIELD E



$$\Delta V_{AB} = \Delta U_{AB} / q$$

The electrical potential difference between A and B equals the work per unit charge necessary to move a charge +q from A to B

$$\Delta V_{AB} = V_B - V_A = -W_{AB}/q = -\int \underline{E}.\underline{dl}$$

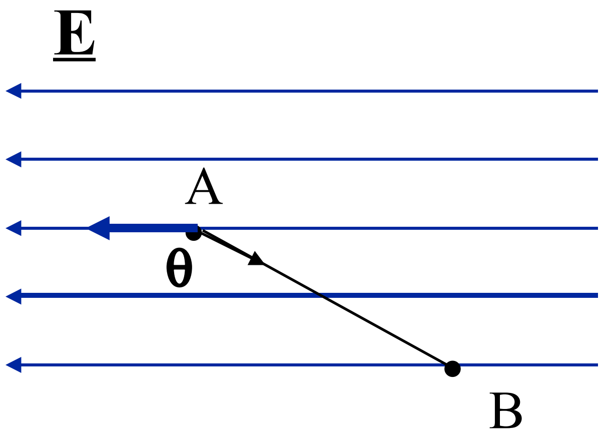
But E = constant, and $\underline{E}.\underline{dl} = -1\ E\ dl$, then:

$$\Delta V_{AB} = -\int \underline{E}.\underline{dl} = \int E\ dl = E \int dl = E\ L$$

$$\Delta V_{AB} = E\ L$$

$$\Delta U_{AB} = q\ E\ L$$

Cases in Which the Electric Field $\underline{E}$ is not Aligned with $\underline{dL}$



$$\Delta V_{AB} = -\int_A^B \underline{E}.\underline{dl}$$

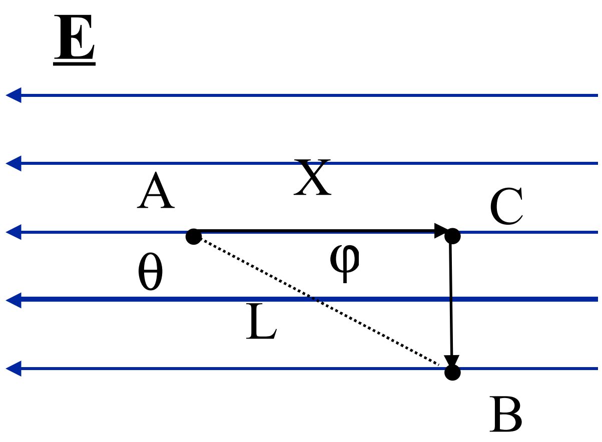
Since $\underline{F} = q\,\underline{E}$ is conservative, the field $\underline{E}$ is conservative.
Then, the electrical potential difference does not depend on the integration path.

One possibility is to integrate along the straight line AB. This is convenient in this case because the field E is constant, and the angle θ between E and dL is constant.

$\underline{E}\,.\,\underline{dl} = E\,dl\cos\theta \Rightarrow$

$$\boxed{\Delta V_{AB} = -E\cos\theta\int_A^B dl = -E\,L\cos\theta}$$

Cases in Which the Electric Field E is not Aligned with dL



$$\Delta V_{AB} = -\int_A^B \underline{\mathbf{E}} \cdot \underline{\mathbf{dl}}$$

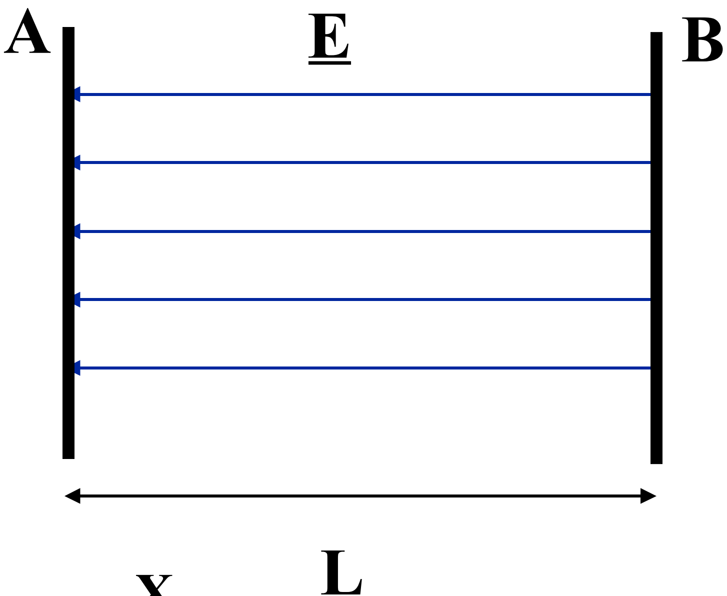
Another possibility is to choose a path that goes from A to C, and then from C to B

$\Delta V_{AB} = \Delta V_{AC} + \Delta V_{CB}$ $\quad \Delta V_{AC} = E\,X$ $\quad \Delta V_{CB} = 0$ ($E \perp dL$)

Thus, $\Delta V_{AB} = E\,X$ $\quad$ but $X = L \cos\varphi = -\,L \cos\theta$

$$\boxed{\Delta V_{AB} = -\,E\,L\cos\theta}$$

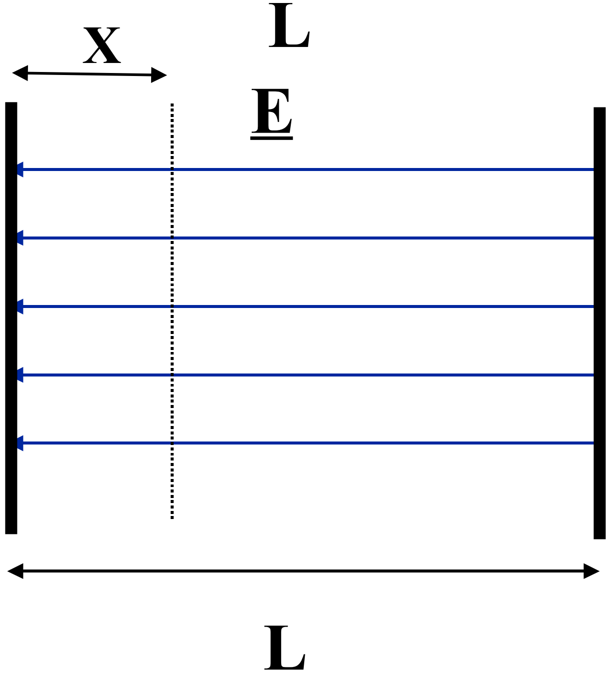
Equipotential Surfaces (lines)



Since the field E is constant

$$\Delta V_{AB} = E\,L$$

Then, at a distance X from plate A

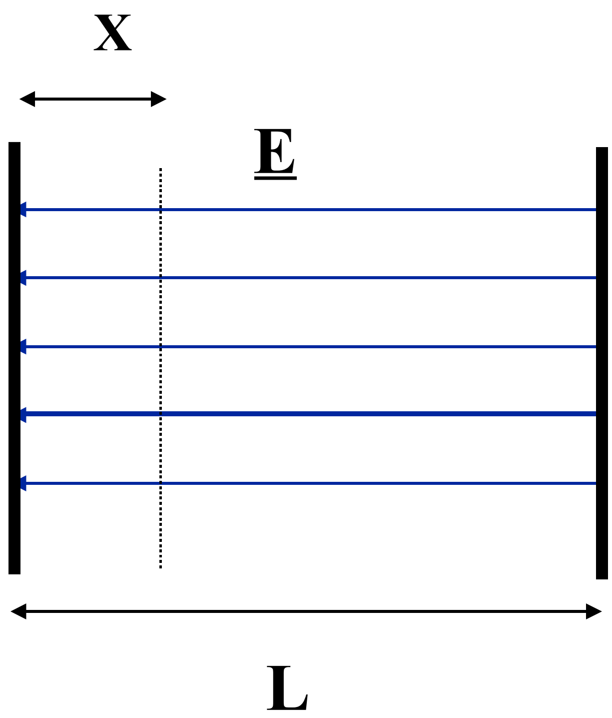


$$\Delta V_{AX} = E\,X$$

All the points along the dashed line, at X, are at the same potential.

The dashed line is an equipotential line

Equipotential Surfaces (lines)



It takes no work to move a charge at right angles to an electric field

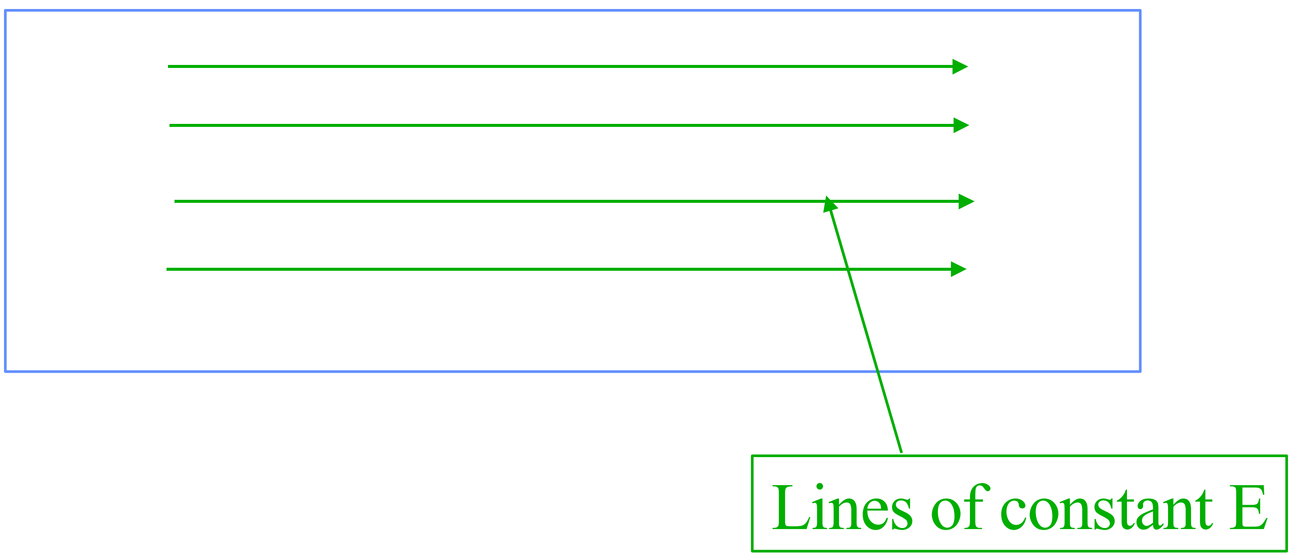
$$\underline{E} \perp \underline{dL} \Rightarrow \int \underline{E} \bullet \underline{dL} = 0 \Rightarrow \Delta V = 0$$

If a surface(line) is perpendicular to the electric field, all the points in the surface (line) are at the same potential. Such surface (line) is called **EQUIPOTENTIAL**

EQUIPOTENTIAL ⊥ ELECTRIC FIELD

Equipotential Surfaces

We can make graphical representations of the electric potential in the same way as we have created for the electric field:



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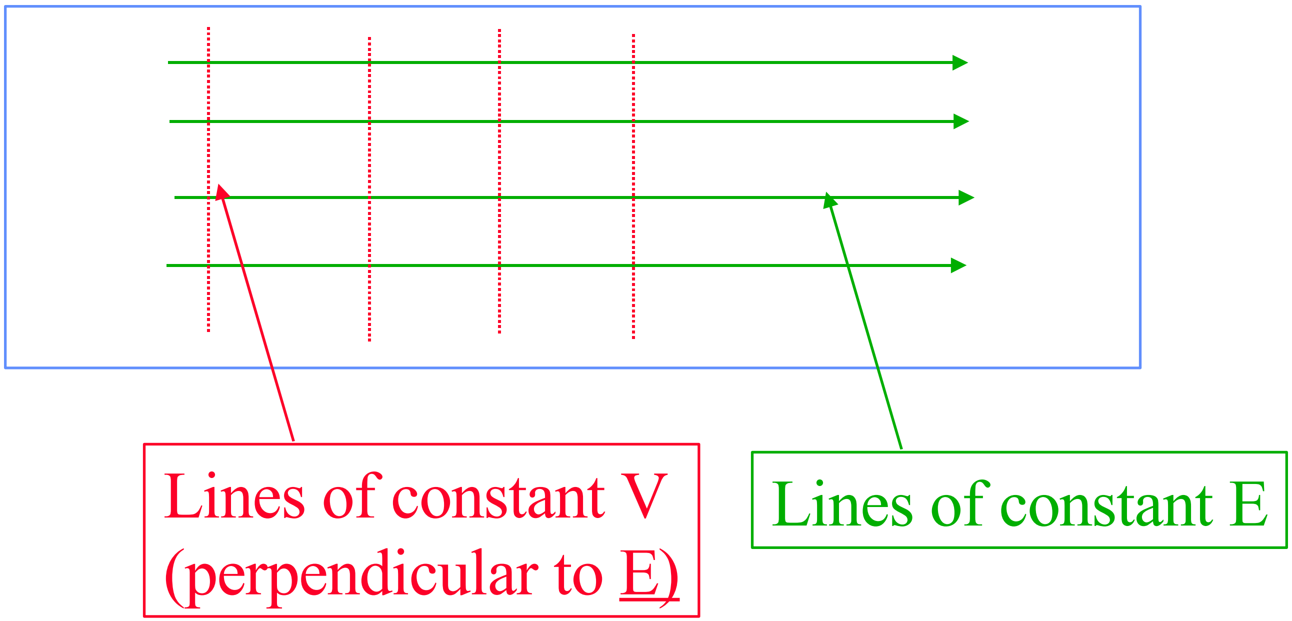

Equipotential Surfaces

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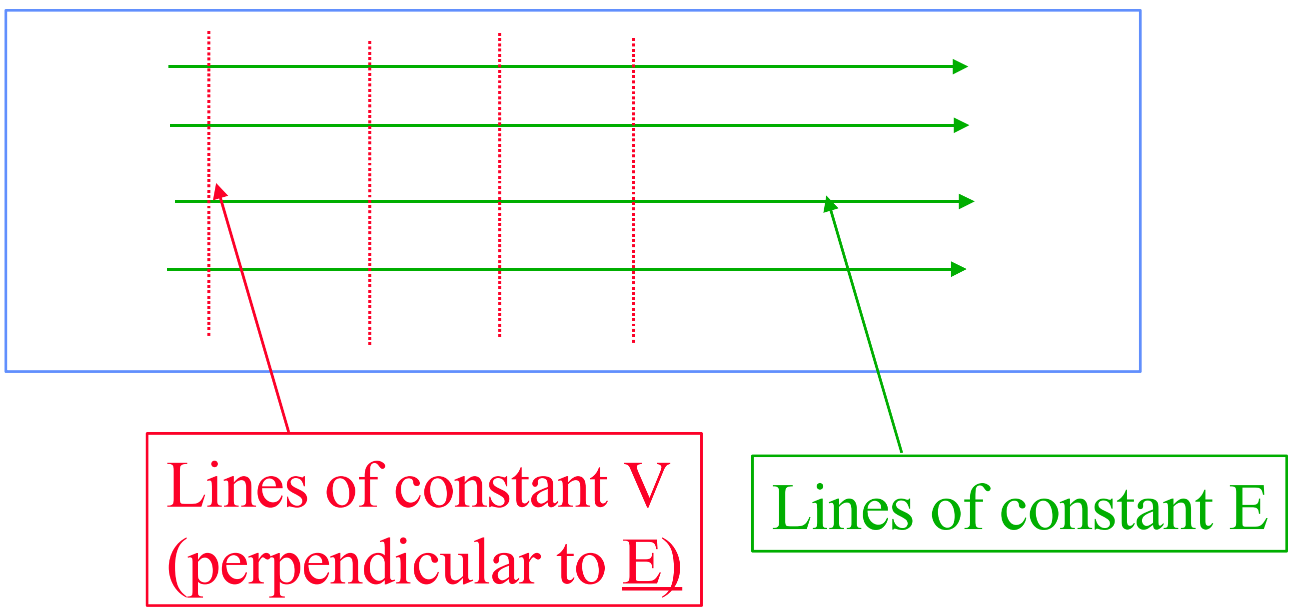
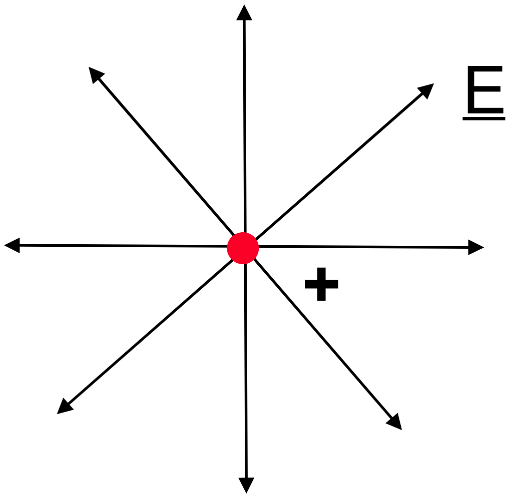


Equipotential plots are like contour maps of hills and valleys.

Equipotential Surfaces

How do the equipotential surfaces look for:

(a) A point charge?



(b) An electric dipole?

Equipotential plots are like contour maps of hills and valleys.