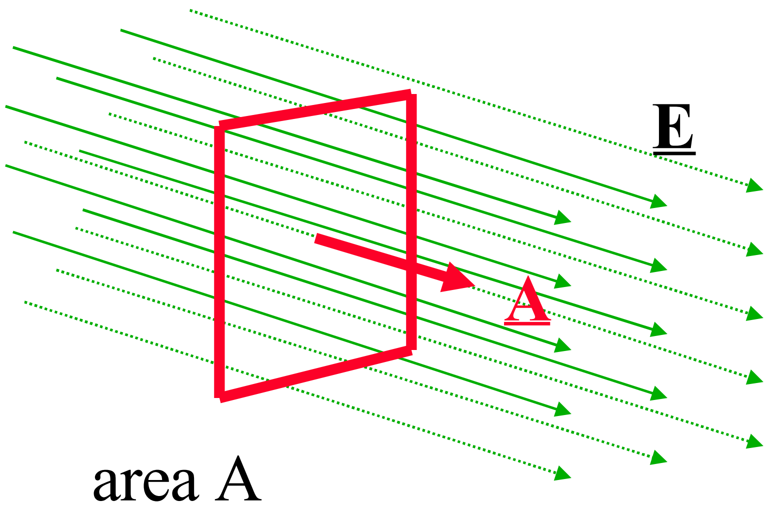
Gauss’s Law

Chapter 24

Electric Flux

We define the electric flux Φ, of the electric field E, through the surface A, as:

$$\Phi = \underline{\mathbf{E}} \cdot \underline{\mathbf{A}}$$

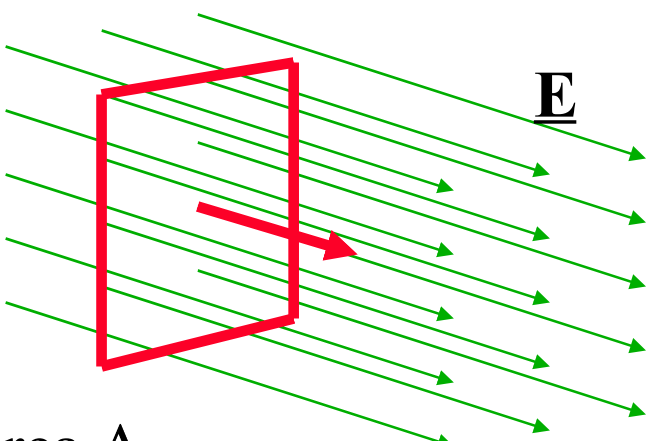


Where A is a vector normal to the surface (magnitude A, and direction normal to the surface – outwards in a closed surface)

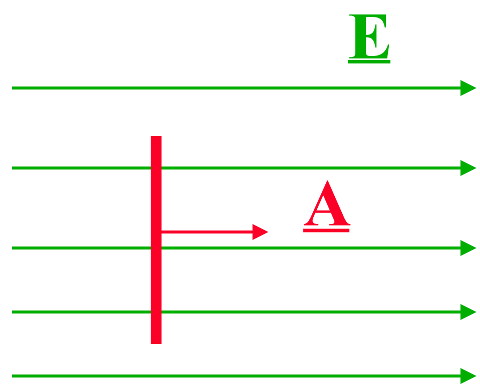
Electric Flux

You can think of the flux through some surface as a measure of the number of field lines which pass through that surface.

Flux depends on the strength of **<u>E</u>**, on the surface area, and on the relative orientation of the field and surface.



area A



Normal to surface, magnitude A

Here the flux is

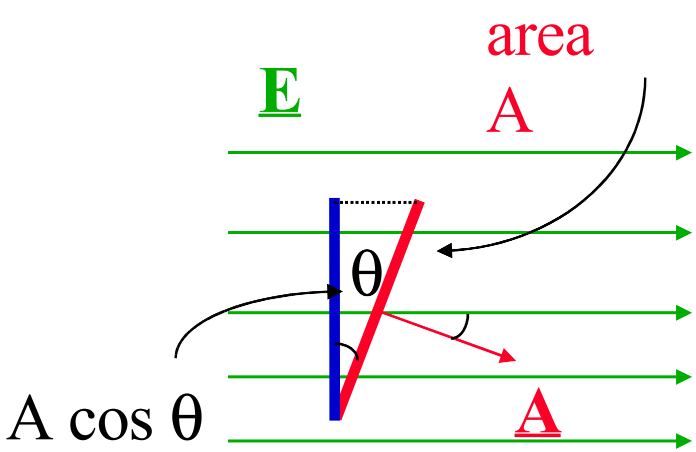
$$\Phi = \underline{\mathbf{E}} \cdot \underline{\mathbf{A}}$$

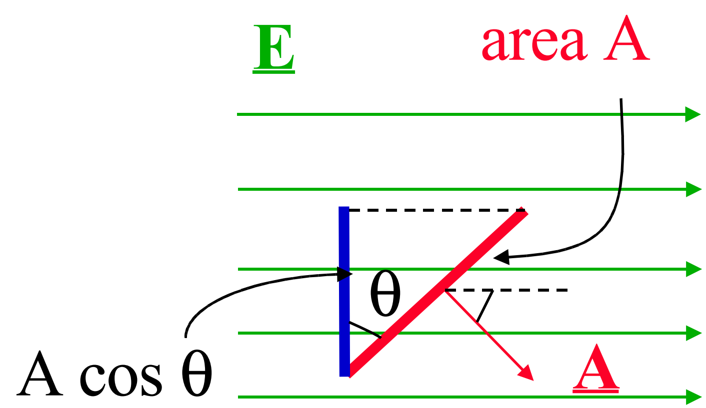
$$\underline{\mathbf{A}} \equiv \longrightarrow$$

Electric Flux

The flux also depends on orientation

$$\Phi = \underline{\mathbf{E}} \cdot \underline{\mathbf{A}} = \mathbf{E\ A\ cos\ \theta}$$

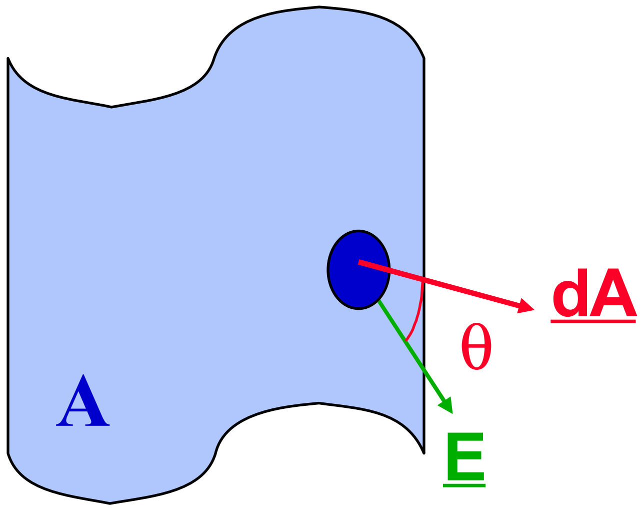




The number of field lines through the tilted surface equals the number through its projection. Hence, the flux through the tilted surface is simply given by the flux through its projection: E (A cosθ).

What if the surface is curved, or the field varies with position ??

$$\mathbf{\Phi} = \underline{\mathbf{E}} \,.\, \underline{\mathbf{A}}$$



1. We divide the surface into small regions with area dA

2. The flux through dA is

$$d\Phi = E\, dA \cos\theta$$

$$\mathbf{d\Phi} = \underline{\mathbf{E}} \,.\, \mathbf{d}\underline{\mathbf{A}}$$

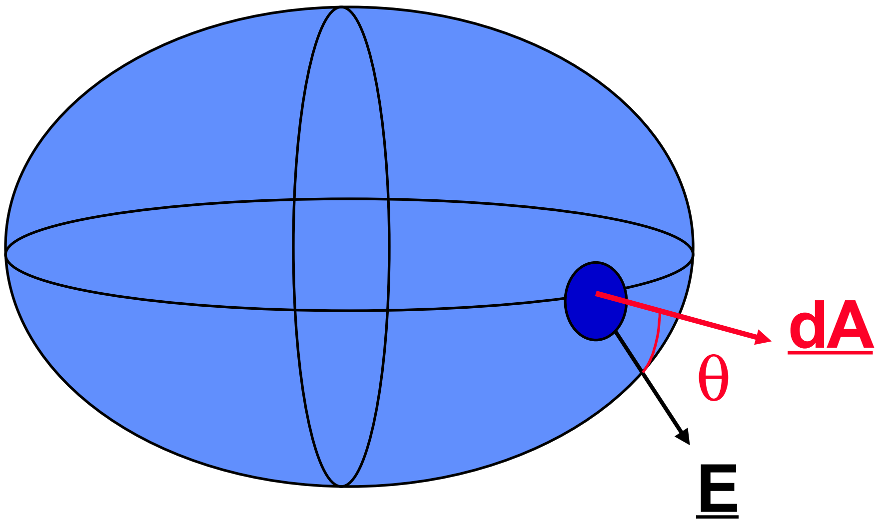
3. To obtain the total flux we need to integrate over the surface A

$$\Phi = \int \mathbf{d\Phi} = \int \underline{\mathbf{E}} \,.\, \underline{\mathbf{dA}}$$

In the case of a closed surface

$$\Phi = \oint d\Phi = \oint \underline{E} \bullet \underline{dA}$$

The loop means the integral is over a closed surface.



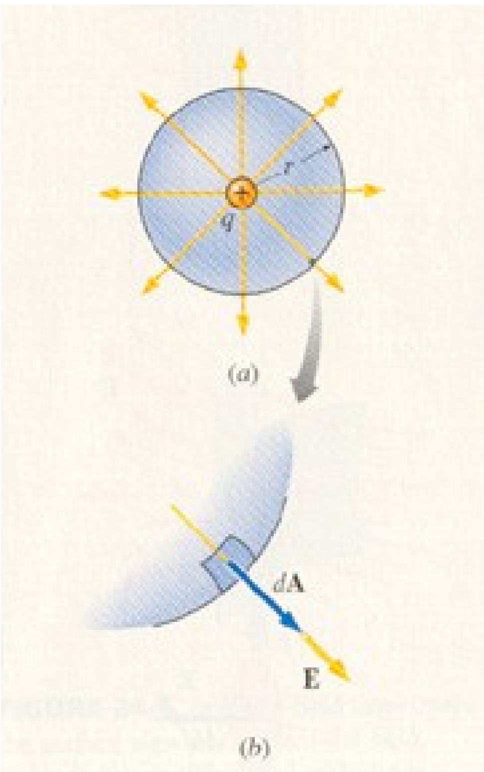
Gauss's Law

The electric flux through any closed surface equals ⩾ enclosed charge / ε_0

$$\oint \underline{E} \bullet \underline{dA} = \frac{\sum_{inside} q}{\varepsilon_0}$$

This is always true. Occasionally, it provides a very easy way to find the electric field (for highly symmetric cases).

Calculate the electric field produced by a point charge using Gauss Law



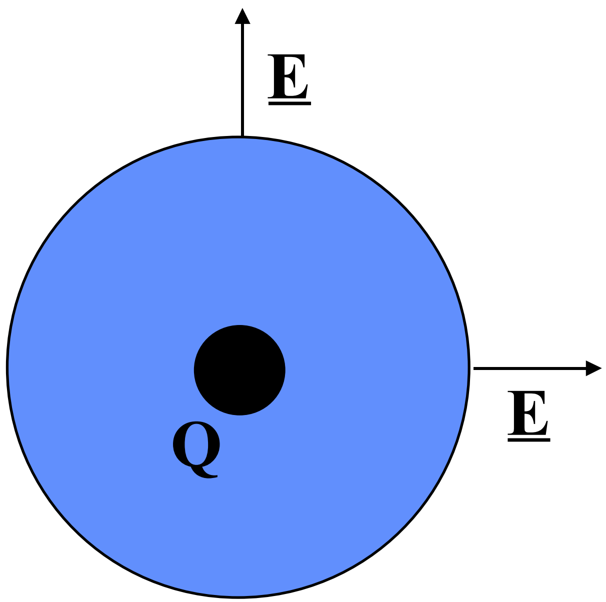
We choose for the gaussian surface a sphere of radius **r**, centered on the charge **Q**.

Then, the electric field **E**, has the same magnitude everywhere on the surface (radial symmetry)

Furthermore, at each point on the surface, the field **E** and the surface normal **dA** are parallel (both point radially outward).

$$\mathbf{E} \cdot \mathbf{dA} = E\,dA \quad [\cos\theta = 1]$$

Electric field produced by a point charge



$$\int \underline{E} \cdot \underline{dA} = Q / \varepsilon_0$$

$$\int \underline{E} \cdot \underline{dA} = E \int dA = E\,A$$

$$A = 4 \pi r^2$$

$$E\,A = E\;4 \pi r^2 = Q / \varepsilon_0$$

$$E = \frac{1}{4\pi \varepsilon_0} \frac{Q}{r^2}$$

$k = 1 / 4 \pi \varepsilon_0$
ε_0 = permittivity
$\varepsilon_0 = 8.85 \times 10^{-12}\ C^2/Nm^2$

→ ***Coulomb's Law !***

Is Gauss's Law more fundamental than Coulomb's Law?

- No! Here we derived Coulomb's law for a point charge from Gauss's law.
- One can instead derive Gauss's law for a general (even very nasty) charge distribution from Coulomb's law. The two laws are equivalent.
- Gauss's law gives us an easy way to solve a few very symmetric problems in electrostatics.
- It also gives us great insight into the electric fields in and on conductors and within voids inside metals.

GAUSS LAW – SPECIAL SYMMETRIES

	SPHERICAL (point or sphere)	CYLINDRICAL (line or cylinder)	PLANAR (plane or sheet)
CHARGE DENSITY	Depends only on radial distance from central point	Depends only on perpendicular distance from line	Depends only on perpendicular distance from plane
GAUSSIAN SURFACE	Sphere centered at point of symmetry	Cylinder centered at axis of symmetry	Pillbox or cylinder with axis perpendicular to plane
ELECTRIC FIELD E	E constant at surface $E \parallel A$ - $\cos\theta = 1$	E constant at curved surface and $E \parallel A$ $E \perp A$ at end surface $\cos\theta = 0$	E constant at end surfaces and $E \parallel A$ $E \perp A$ at curved surface $\cos\theta = 0$
FLUX Φ			

Applying Gauss's law in spherical geometry